THE
GAUGER'S GUIDE;
OR,
Exciſe-Officer Inſtructed.

CONTAINING,

I. A copious and ample Treatiſe of Decimal Arithmetic, wherein are many Things not commonly known, and adapted to the Buſineſs of Gauging.

II. The Extraction of the Square or Cube-Root of whole or mixt Numbers, and of Vulgar and Decimal Fractions, with the Demonſtration and Reaſon of the myſterious Proceſs.

III. The grand and principal Foundation of Gauging; the Menſuration of Superficies and Solids, regular and irregular, after the beſt Manner, agreeable to the ſubſequent Matter.

IV. As a farther Introduction to the Knowledge of Gauging, here are, 1. The Way to find Proportions of and Mean Proportions between Numbers Arithmetical and Geometrical. 2. Tables of Beer, Ale, Wine and Malt-Meaſure, with the ſolid Inches therein. 3. Of Gauge-Points, their Calculation and Uſe. 4. The Reduction of Meaſures from one to another, ſeveral Ways. 5. Tables of Beer, Ale and Wine-Meaſure in any Number of ſolid Inches.

V. The whole Art of Gauging. 1. Coppers and other open Veſſels cylindrical or conical, right-lined or curved, with plain and eaſy Rules and Examples of inching the ſame. 2 The Gauging of Casks which lie with their Axis parallel to the Horizon, ſix or ſeven ſeveral Ways; and how to find the Ullage and Content when part full, with a Table of the Areas of Segments of a Circle, the Uſe and Calculation thereof, and how to find the Ullage or Content without ſuch Table, in a new Way. 3. A new Method to find the Ullage or Content of Liquor remaining in Casks that ſtand on one End, with their Diameters parallel to the Horizon, and an Example of inching the ſame. 4. Tables of the Areas of Circles in Beer, Ale and Wine-Gallons, and their Uſe.

VI. How to Gauge Malt in the Ciſtern, Couch and Floor.

To all which is added,

A SUPPLEMENT of the Duties of Exciſe.

Tables I, and II, X, XI and XII, to be paid by common Brewers; III and IV. by Victuallers; V, and VI. for Malt; VII. for Plate; VIII. for Soap; IX. for Candles and for Printed Linnen, with the Uſe of each; and how to calculate the Allowances in four of the ſix firſt, and three laſt Tables, viz. To common Brewers and Malſters. The moſt Copious Set of Tables, for the Duties of Exciſe, yet publiſhed.

By E. HATTON, *Philomath.*

The whole approved of and recommended by long experienced Officers, who have been and are Supervisors, Surveyors, Examiners and Collectors; as by their own Hands ſubſcribed appears. *The like not Extant.*

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BRITISH MUSEUM

TO

The Right Honourable

Sir *Robert Walpole*,

Firſt COMMISSIONER
of the TREASURY;

CHANCELLOR and UNDER-TREASURER
of the EXCHEQUER;

One of His MAJESTY'S Moſt Honourable
PRIVY-COUNCIL;

AND

KNIGHT of the Moſt Noble
ORDER of the GARTER:

This TREATISE is moſt humbly dedicated,

By the AUTHOR.

BRITISH MUSEUM

THE PREFACE.

IT is now near 40 Years ſince I firſt apply'd my Thoughts to the Study of what is the Subject of this Treatiſe; ſoon after which, I made ſuch Proficiency as to inſtruct ſome who paſſed the Examination of being qualify'd for the Exciſe, and have been many Years approved Officers, Supervisors, &c. *But other Buſineſſes having employ'd me for above 30 Years laſt paſt, I had no Thought of writing on the Subject of Gauging, altho' I have publiſh'd ſeveral Tracts relating to Numbers, Species, Surveying, Architecture, Heraldry,* &c. *And having for ſome Months paſt had a Relaxation from other Avocations, I was put upon writing on this Subject, in which I have taken ſome Care and Pains to make it more eaſy, copious and uſeful than what I have obſerved to be in print; (and I think I have look'd into moſt, if not all that are extant.) In order to which, I have firſt treated on ſuch Things as are to be previouſly known, to enable any one to acquire the Knowledge of Gauging with more Facility: As* Decimal Arithmetic, *which is here fully treated of, and contains many Things not commonly known, (too many to enumerate,) which I doubt not but will be very pleaſing and acceptable to the Readers, eſpecially thoſe deſigned for the Exciſe, tho' they may have made good Progreſs in Decimals before.*

I have next given plain and eaſy Rules (*ſuch as naturally reſult from Algebraic Canons*) *for the* Extraction of the Square and Cube Roots *of whole or mixt Numbers, and of Vulgar and Decimal Fractions, and have given the Reaſon and Demonſtration of the ſeveral Steps in the whole myſterious Proceſs, which the Reader may conſider afterward at pleaſure.*

Then I have in Chap. III. *ſhewn the* Menſuration of all kinds of Superficies and Solids, *as being the grand and ſubſtantial Foundation, whereon the Art of Gauging is built, and which, I believe will be found done after the beſt Manner to anſwer the End of what is ſubſequent.*

Chap. IV. *contains a yet nearer Approximation in the Nature of its Matter to what is principally intended; for here are ſeveral Ways of finding* Proportions to, *and* Mean Proportions, *both arithmetical and geometrical between Numbers. 2dly,* Tables of Beer, Ale, Wine *and* Malt Meaſure, *with the ſolid Inches in each Denomination. 3dly, How* to find all manner of Gauge-Points, *with their Nature and great Uſe in diſcovering the Areas of Circles and Inching of Veſſels. 4thly, The* Reduction of one kind of Meaſure to another, *ſeveral Ways. 5thly,* Tables of Beer, Ale, and Wine-Meaſure, *in any Number of* ſolid Inches; *which may be of good Uſe both to ſuch as do not well underſtand Diviſion, and to prove an Operation for thoſe that do; eſpecially where it is only required to find the whole Content of a Veſſel without Tables of the Areas of Circles in Liquid-Meaſure.*

In the V*th Chapter is taught the whole Art of Gauging; for* (*beſides the Content of any Depth*

Depth of Cylinders and Parallelopipeds) there are very plain, copious and manifeſt Rules to gauge and inch the Fruſtum of a Pyramid, both as ſtanding on its greater and leſſer Baſis, with Examples of what every Inch in Depth contains, and the Way to make the Tables thereof: How to gauge and inch an elliptical and conical Cooler; with Examples of the Content of each Inch, and the Calculation of the Preparatory Scheme and the whole Table: To gauge and inch a large Copper, with two Examples of meaſuring the Crown; and Tables ſhewing what every Inch does contain, how to make the ſame, and the Preparatory Scheme.

In Cask-gauging (*the moſt abſtruſe and curious Branch of this Art, I have inſerted many Things which are not only* intirely new, *but eaſy and perſpicuous to a mean Capacity; and this, whether the Situation be with the Axis or Length, Parallel, or Perpendicular to the Area whereon it is placed; and have ſhewed five Ways here, beſides others afterwards, (three of which are my own,) of finding the whole Content, as the Middle Fruſtum of a Spheroid. And having alſo, in Figure marked* E. *deſcribed the whole Spheroid (which as a Solid is meaſured under Prop.* 8. *Sect.* 2. *of Chap.* III.) *alſo the Middle Fruſtum and Mean Diameter; which laſt I have ſhewed is not to be found (truely) by multiplying the Difference of Diameters by* .7, &c. (*which gives in the Example* .446 *of a Gallon too little) but by* .70342, &c.

I have given you, in the next Place, an Explanation of the Lines and Parts of a Circle, *ſo far as relates to Gauging, (as by the Figure* *mark'd* F.) *and likewiſe a* Table of the Areas of the Segments *of a Circle, and* its Uſe *in the common Way of finding the Ullage (or Wants of Liquor) in any Cask in a lying Poſition, or what remains therein when part is drawn off; which Table I have carefully examined and corrected in ſeveral Places, ſo that the ſame may now be depended on as accurate. I have then given two Rules to prove the Truth of the Method inſerted for finding the Ullage of or Liquor in a Cask; the firſt* by the Mean Diameter, *the ſecond by* comparing *the Contents upon the* wet and dry Inches. *But that which is chiefly to be inſiſted on in this Matter of Ullages of lying Casks, is the third Rule, which is* wholly new, *and wherein I have proceeded* gradatim *to ſhew how to* find the Liquor remaining *in a Cask that is partly filled,* without the Table of the Areas of Segments; the Calculation of which Table *I have alſo ſhewed, and have given an Example, (the ſame with that of the beſt Author now extant,) and I appeal to the Reader if my Method is not much eaſier; and I am aſſured 'tis more accurate (by the Indication of Fact) my Area of the Segment being true to the 4th Decimal Place (as appears by the Table) which his is not.*

Then as to Casks ſtanding on one End (whoſe whole Content may be found as thoſe in a lying Poſition) I have ſhewed a new, eaſy and moſt natural Way of finding the Quantity of Liquor or Ullage (which I admire was never thought on before) and that is by ſuppoſing a Segment in the Middle of, and which will be ſimilar to the Fruſtum of the Spheroid given, which will have one and the ſame Bung-Diameter: i. e. *If the Surface* *of the Liquor in the given Cask, be above the Bung-Diameter, I take as much below that Diameter which will form a Segment or leſſer Fruſtum of a Spheroid in the Middle of that given; and its Content I find by the ſame Rule as the whole Cask; to half of which, I add the Content of the leſſer Cask, and the Sum is the Quantity of Liquor required, when the given Cask is more than half full.*

Or if the given Cask be leſs than half full, then I ſet ſo much above the Bung-Diameter, as the Surface of the Liquor is ſhort of it, and finding the Content of that inner and vacant Segment by the Rules for the Content of the whole Cask, I deduct half the leſſer from half the greater, and the Remainer is the Liquor in the Cask given, if leſs than half full; all which is explained by the Figure marked M. *And then follow Rules and an Example of the Content of every Inch deep of ſuch a Cask ſtanding on one End, and how to calculate the ſame and the Preparatory Scheme.*

I have next given ſome Directions concerning gauging Wine and Oil in the Port of London, *with ſome Uſes of the Diagonal Line, and the Application of the* 47.1 Euclid, *in finding the Diameter, Length and Diagonal Inches of a Cask, by having any two given; and then I have cloſed the gauging of Liquids with two very accurate and uſeful Tables of the Areas of Circles in Beer, Ale or Wine-Gallons, and their Uſe, which I have fully exemplified beyond what is uſual: And laſtly, I have given ſufficient, tho' brief Directions, for gauging Malt in the Ciſtern, Couch and Floor.*

I have farther, by way of Supplement, exhibited Tables both accurate and uſeful, of the Duties to be paid by Common Brewers, (both without and within the Weekly-Bills,) Victuallers, Malſters, Silver-Smiths, Soap-Boilers, Tallow and Wax-Chandlers, Callico and Linnen-Printers; for all of whom, as well as for thoſe that charge and receive the Duties of Exciſe, this Part will be very neceſſary; the like both for Variety and Copiouſneſs being never publiſhed, that I could find. And where there is any Difficulty (from the Allowances) I have ſhew'd how to calculate the Tables; as where the Allowance is 3 in 23 Barrels of Beer, and 2 in 22 Barrels of Ale within the Bills of Mortality, or 2 and a half in 23 Barrels for Strong and Small-Beer, and Ale without the ſaid Bills; and alſo the Calculation of the Allowances and Duty to be paid for Malt, whether the ſame may have been leſs, or 30 Hours or more out of the Ciſtern.

But before I mention'd the Supplement, I ſhould have acquainted the Reader that I have purpoſely omitted Operations by the Rule or Inſtrument, as not admitting of ſufficient Verity; ſo either the Revenue or the Subject muſt ſuffer by that Gueſs-way of working: beſides, it carries nothing of Reaſon or Demonſtration along with it; and I wonder it hath never been ſo repreſented: The Method by the Pen, and proper Tables to be calculated on all Occaſions, being much more demonſtrative, accurate, eaſy and expeditious; but of this I have much more to ſay on another Occaſion.

I The Underwritten, who have been near 40 Years in the Exciſe, (Gauger, Superviſor and Collector) having peruſed this Book, do entirely approve of and recommend the ſame, as the beſt I have ſeen; there being many Things in the ſeveral Parts which are wholly new, and 'tis done with Judgment, Care and Pains, Perſpicuity and Accuracy, for which the Author deſerves the Thanks and Favour of the Publick, as well as for his other Books, which he hath publiſhed for the Benefit of Mankind in general, and his Country in particular.

THO. POOLE.

I Do alſo very highly approve and recommend this Book, as the moſt copious, accurate and uſeful that hath been done on the Subject; not doubting but the judicious and impartial Reader, will, to his great Satisfaction, find it to anſwer the Preface and the Character given of it as above, and even to exceed his Expectation.

RICHARD HAZARD,
Surveyor and Examiner.

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Decimal Arithmetic.

THere are Four Things neceſſary to be taught before the Art of Gauging, as being uſed in acquiring the Knowledge, and in the Practice thereof; *i.e.*

1. Decimal Arithmetic.
2. Extraction of the Square and Cube Roots.
3. The Menſuration of Superficies and Solids. And,
4. Inſtructions as to Liquid and Dry Meaſures, Gauge-Points, *&c.* Of which I ſhall proceed to Diſcourſe and Explain in the moſt eaſy and natural Method I can, and in that Order.

CHAP. I.

Of Decimal Arithmetic.

NUmber is the Subject of all Arithmetic, and (to ſay nothing of Artificial or Logarithms, as not of moment in Gauging) may be properly diſtinguiſhed into whole or broken Numbers; and as the former are infinite

Units from one incluſive upward; ſo the latter (or Fractions) are Parts of a Unit, which are alſo infinite.

Fractions are either Vulgar or Decimal. A Vulgar is a Fraction expreſſing any Parts of a Unit by its Terms.

The Terms are the Numerator and Denominator, placed under and over a ſmall Line, thus,

$$\left.\begin{array}{l}\underline{3}\text{ the Numerator}\\ 5\text{ the Denominator}\end{array}\right\}\text{three fifths.}$$

The Denominator does name the Number of Parts into which the Unit is divided in the Fraction given, and the Numerator ſhews the Number of thoſe Parts contained in the Fraction, as the 5 above doth indicate, that the Unit is divided into 5 equal Parts; and the 3 tells you, that 3 of thoſe fifth Parts is expreſſed by the Fraction; and is read 3 fifths: So alſo $\frac{5}{7}$ is 5 ſevenths; $\frac{8}{11}$ is 8 elevenths; $\frac{21}{112}$ is 21 one hundred and twelfth Parts; and ſo is any other Vulgar Fraction wrote and read. But,

Decimal Fractions have no Numbers for their Denominator, except 10 or ſome Power of 10, as 100, 1000, *&c.* from whence the Fraction has its Name *Decimal.* Another thing wherein this Fraction differs from the Vulgar Fraction, is its being commonly wrote without its Denominator, by putting a point towards the left hand of the Numerator, over the place where the 1 would be, if the Denominator were wrote down: Thus $\frac{5}{10}$ is wrote .5, $\frac{15}{100}$ is .15; $\frac{125}{1000}$ is .125; and $\frac{57}{1000}$ is .057, a

Cypher being put in the laſt, to make the Point in the thouſands place, ſhewing the Unit to be divided into 1000 Parts, or that the Denominator, if wrote down, is 1000; as in this Table.

$\frac{1}{10}$ is wrote .1, and read one tenth.
$\frac{1}{100}$ ——.01, one hundredth part.
$\frac{1}{1000}$ ——.001, one thouſandth part, &c.
$\frac{12}{100}$ —— .12 twelve hundredth parts.
$\frac{12}{1000}$ —— .012 twelve thouſandth parts.
$\frac{123}{1000}$ —— .123 —— 123 thouſandth parts.
$\frac{123}{10000}$ ——.0123 ——123 ten thouſand parts.
$\frac{123456}{1000000}$ —.123456 —123456 Million parts, &c.

Having thus ſhewed how to read and write a Decimal, the next thing is,

Sect. 2. *Reduction of Decimals.*

THIS conſiſts of 2 Parts, *i. e.* either, I. The reducing Money, Weight, Meaſure, &c. of inferior to Decimals of a ſuperior Denomination: or,

II. The reducing Decimals of a ſuperior Denomination, to the Parts of Coin, Weight or Meaſure which they contain: Or to find the Value of thoſe Decimals.

I. As Vulgar Fractions may be reduced to others of different Terms, ſo by the ſame Rule may Vulgar be reduced to Decimal, by only aſſigning for the Denominator a Decimal Number; as 1. For as $\frac{3}{4}$ is reducible to a Fraction whoſe Denominator is 16, by this Rule, as 4 to 3 :: ſo 16 to 12; and $\frac{12}{16}$ is equal to $\frac{3}{4}$: ſo may $\frac{3}{4}$ be Reduced to a Decimal whoſe Denominator is 100; for as 4 to 3 :: ſo 100 to 75; or prefixing a Point, is .75. So that to find the Decimal of any Coin, Weight or Meaſure, you have nothing to do but firſt to reduce ſuch Coin, *&c.* to the Vulgar Fraction of the higher Denomination required: And then by the Proportion above to reduce that Vulgar Fraction to a Decimal.

Thus to reduce 1 (or any Number of Shillings) to a Decimal; I ſay, that in a Vulgar Fraction 1 *s* is $\frac{1}{20}$ of a Pound: therefore to reduce that to another Fraction, whoſe Denominator is 100 (or to a Decimal) I ſay as above, as 20 to 1 :: ſo 100 to .05 *l.*

You muſt in this Caſe cut off ſo many in the Quote for Decimals, as there are Cyphers in your third Number (or that next the right hand;) and if there be not ſo many Places, you muſt put 1 or more Cyphers towards the left hand, as in this laſt Example. So for the Decimal of 3, 4, 5, or any Number of Shillings, ſay as in the laſt;

As 20 to 3 :: ſo 100 to .15 = the Decimal of 3 *s.*
As 20 to 4 :: ſo 100 to .20 = the Decimal of 4 *s.*
As 20 to 5 :: ſo 100 to .25 = Decimal of 5 *s.* *&c.*

Note, that a Cypher next to the right hand of any Decimal may be omitted.

2. So alſo for the Decimals of Pence: as ſuppoſe 7 *d.* there being 240 Pence in 20 *s.* I therefore ſay 7 *d.* is $\frac{7}{240}$ in a Vulgar Fraction: And I ſay, As 240 to 7, ſo 1000 to .029 = the Decimal of 7 *d.* *&c.*

3. For the Decimal of a Pound, which any Number of Farthings are; conſider that 960 Farthings make 1 *l.* therefore, ſuppoſe for 3 Farthings which are $\frac{3}{960}$ of a Pound,

As 960 to 3 :: ſo 10000 to .0031

4. *A ſecond Way for theſe Reductions*; as ſuppoſe to reduce any Number of Shillings to Decimals of a Pound. The Decimal of 1 *s.* being .05, multiply that by any Number of Shillings, gives the Decimals of thoſe Shillings; as,

	.05	.05	.05	.05	.05
Mult. by	2	by 3	by 4	5	by 6
Decim. of 2 *s.*	.10	.15, of 3 *s.*	.20, of 4 *s.*	.25, of 5 *s.*	.30 Dec. of 6 *s.*

5. And the Decimal of 1 Penny being found by the Rule above to be .004166, that multiplied by 2 gives .008333 = the Decimal of 2 *d.* by 3 gives .0125 the Decimal of 3 *d.* *&c.*

6. *A third Way to reduce* Shillings, *&c.* to Decimals of a Pound. For Shillings that are an even Number, take half, is the Decimal

of a Pound, and ſo is half an odd Number with a Cypher added to the Decimals.

Of 1 *s.* is .05	6 *s.* is .3	11 *s.* is .55	16 *s.* is .8
2 — .1	7 — .35	12 — .6	17 — .85
3 — .15	8 — .4	13 — .65	18 — .9
4 — .2	9 — .45	14 — .7	19 — .95
5 — .25	10 — .5	15 — .75	

7. So alſo for Pence; the Decimal of 1 *s.* being = .05, half that is the Decimal of 6 *d.* = .025; a fourth of .05 is = .0125 the Decimal of 3 *d.* a 6th of .05 is .008333 the Decimal of 2 *d.* and half that is .0041666 = the Decimal of a Penny; more .0125 (the Decimal of 3 *d.*) is = .0166666 = the Decimal of 4 *d.* and the Decimals of 4 *d.* and 3 *d.* is the Decimal of 7 *d.* of 4 *d.* doubled is the Decimal of 8 *d.* of 6 *d.* and 3 *d.* is the Decimal of 9 *d.* of 5 *d.* doubled is the Decimal of 10 *d.* of 6 *d.* and 5 *d.* is the Decimal of 11 *d.* Alſo a 4th of the Decimal of a Penny is = .0010416 the Decimal of a Farthing; twice that or .0020832 = the Decimal of 2 Farthings, and 3 times the Decimal .0010416 is .0031248 = the Decimal of 3 Farthings.

So that .0010416 is the Decimal of 1 *qr.*
.0020832 — — of 2
.0031248 — — of 3
.0041666 — — 1 *d.*
.0083333 — — 2
.0125 — — — 3
.0166666 — — 4
.0208333 (or that of 3 *d.* and 2 *d.*) of 5

.025	is the Decimal of	6
.0291666	—— ——	7
.0333333	—— ——	8
.0375	—— ——	9
.0416666	—— ——	10
.0458333	—— ——	11

8 *A fourth Way to reduce Money* of a lower Denomination to Decimals of a higher, which is the eaſieſt, beſt, and moſt accurate, eſpecially where the Denominations are ſeveral: as, to reduce 19 *s.* 11 *d.* $\frac{3}{4}$ to the Decimal of a Pound.

s. *d.*
*l.*9989583 --- = 19 : $11\frac{3}{4}$ in the Decim.
Diviſors. (of a Pound.
20) *s.* 19.9791666 --- = — : $11\frac{3}{4}$ in the Decim.
(of a Shill.
12) *d.* 11.7500000 --- = — : — $\frac{3}{4}$ in the Decim.
(of a Penny.
4) *q.* 3.00 ; begin to divide here by 4 ; then divide 11.75 &c. by 12, as tho' all were Integers, and then divide that Quotient by 20, taking into it the 19 *s.* as the 11 *d.* into the Dividend preceding.

9. *There is alſo a fiſth Way* of reducing Money of any lower Name to Decimals of a Pound ; done by help of the little Tables above: as the Decimal of 19 *s.* $11\frac{3}{4}$ is.

Of 19 *s.* as by the Table under the ſixth Rule above is = } .95
Of 11 *d.* as by the laſt Table = .0458333
Of 3 *qr.* as *per* 3d Decimal above = .0031248

Sum = .9989581

The difference in the 1ſt Figure towards the right hand is not conſiderable, being only 2 ten Million parts of a Pound.

10. *Laſtly, there is a ſixth Way* (and this I take to be more by three than any Author has given) to reduce Shillings, Pence, *&c.* to Decimals of a Pound, which Rule is exact enough when the Decimal is not to be multiplyed, and is this: Place half the Number of Shillings in Primes place next the Point; and if there be an odd Shilling, for that place 5 in the ſecond place from the Point: Then for the Pence and Farthings, bring them into Farthings, and put Units place thereof in the third place from the Point, and the Tens in the ſecond place: and as often as the Number of theſe Farthings amount to 12 and upward to 36, add 1, and if 36 or upward add 2.

Exam. 1.	*Exam.* 2.	*Exam.* 3.	*Exam.* 4.	
6*s.* 2*d.* is	9*s.* 3*d.* is	12*s.* 8*d.* $\frac{1}{4}$ is	19 : 11$\frac{3}{4}$ is	
= .308	= .4	= .634 *l.*	= .9	Sum
	5		.05	= .999 *l.*
	13		.049	
	Sum .463 *l.*			

Which laſt Example agrees with that under the 8th and 9th Heads above, within one 25000 part of a Pound; or near $\frac{1}{25}$ of a Farthing.

11. To reduce Averdupois Weight to Decimals; as 1ſt in the little Weight, reduce 12 ℥ 13 ʒ to Decimals of a Pound.

By the 1ſt Rule above (in 12 ʒ. 13 ℥. there being 205 Drams, and 256 Drams in a Pound) the Vulgar Fraction being $\frac{205}{256}$; therefore the Decimal is .8007 ℔. for,

As 256 is to 205 :: ſo 10000 to 8007

12. In the greater Averdupois Weight, what Decimal of 1 hundred weight is 3 *q.* 7 ℔? In 3 *q.* 7 ℔ there are 91 ℔ which is $\frac{91}{112}$ of *C.* Therefore,

As 112 is to 91 :: ſo 10000 to 8125; ſo that the Decimal .8125 is the Anſwer.

13. But that which I think moſt proper for a Book of Gauging, is to inſiſt more fully on the Decimals of liquid and dry Meaſure, which I ſhall not only do, but ſhall alſo give Tables thereof. What Decimal of a Barrel of Beer is two Firkins? One Firkin being $\frac{1}{4}$ of a Barrel, therefore 2 is $\frac{2}{4}$; and,

As 4 to 2, ſo 10 to 5; or .5 of a Barrel the Decimal. So the Decimal of 8 Gallons is .2222, for the Decimal of 1 Gallon is $\frac{1}{9}$ of $\frac{1}{4}$ (or $\frac{1}{36}$) of a Barrel, and 8 muſt of Conſequence be $\frac{8}{36}$; therefore as 36 to 8 :: ſo 10000 to 2222 or .2222 of a Barrel.

Note, Where a Decimal is a repeated Figure, put *r* and 1 after it; or if 2 Figures repeat, as .2525, *&c.* put 25 and *r*2 after. Or if 1, 2 or 3, *&c.* Figures repeat in the ſame Number after other Figures, as 35757, you may write 357*r*2, *i. e.* the two Figures 57 do repeat; and ſo 7345345345, write 7345*r*3, tho' theſe are for the moſt part but 1 repeating Figure, as *a*3, *a*5 or *a*6, of which Repeaters ſpecial Notice muſt be taken (eſpecially in Addition and Multiplication) what the Sums or Products would be of thoſe repeat-

ing,

ing, but not wrote down, that you may know what to carry, as ſhall be ſhew'd in thoſe Rules, *&c.*

Example 3. To reduce 3 Firkins 8 Gallons 3 Pints into the Decimal of a Barrel of Beer. $\overset{\text{F. G. P.}}{3:8:3}$ are 283 Pints, and there being in a Barrel or 36 Gallons, 288 Pints; therefore 283 Points is $\frac{283}{288}$ of a Barrel: For the Decimal of which, I ſay,

As 288 to 283 :: ſo .10000 to .9826 = the Decimal.

14. For reducing Ale-Meaſure into Decimals, you muſt obſerve that the Barrel is but 32 Gallons, or 4 Firkins of 8 Gallons each.

Example 1. What Decimal Part of a Barrel is 5 Gallons? 1 Gallon is $\frac{1}{8}$ of $\frac{1}{4}$ or $\frac{1}{32}$; ſo 5 is $\frac{5}{32}$ of a Barrel, and as 32 to 5, ſo .10000 to .1562. *Example* 2. What is $\overset{\text{F. G. P.}}{1:3:7}$ in the Decimal of a Barrel of Ale? In 1 : 3 : 7 are 95 Pints, and 256 Pints being the Barrel, therefore 95 Pints is $\frac{95}{256}$ Barrel; and as 256 to 95, ſo .1000 to .371 of a Barrel.

15. For dry Meaſure, the Decimals are found after the ſame manner; as ſuppoſe the Decimal of a Buſhel were required, that 3 Pecks 1 Gallon is. There being 2 Gallons in 1 Peck, and 4 Pecks in 1 Buſhel, $\overset{\text{P. G.}}{3:1}$ is 7 Gallons; and there being 8 Gallons in a Buſhel, the Vulgar Fraction is $\frac{7}{8}$; and therefore as 8 to 7 :: ſo .1000 to .875 Parts of a Buſhel, which was required.

Theſe Rules I thought neceſſary for reducing Beer, Ale and Malt, their lower Meaſures into Decimals of higher. As for Tables of their

Quantities of lower contain'd in higher Denominations, their Capacities in ſolid Inches, Gauge-Points, *&c.* I ſhall ſhew them next before the Art of Gauging it ſelf: In the mean time, I proceed with ſhewing,

16. In the next place, that the above Example 3. of Rule 13. may be done after that eaſy and accurate Way under Rule 8. as by theſe 2 Examples.

Decimals *Beer* *F. G. P.*
Bar. .9826 = of a Barrel that 3 : 8 : 3 is
4) Fir. 3.9305*r*1 = of a Firkin that : 8 : 3 is
9) Ga. 8.3750 = of a Gallon that : 0 : 3 is
8) Pt. 3.00 = here begin to divide by 8.

Example 2. *Rule* 14. *done this Way.*
F. G. P.
Bar. .371 of a Barrel of Ale that 1 : 3 : 7 is
4) Fir. 1.4843 of a Firkin that 0 : 3 : 7 is
8) Ga. 3.875 of a Gallon that 0 : 0 : 7 is
8) Pts. 7.000 here begin to divide by 8.

Here follows Beer and Ale-Meaſure in the Decimals of a Barrel of *London* Meaſure.

Beer Meaſure.				Ale Meaſure.			
Fir.	Decimals of a Bar.	Gallons	Decimals of a Bar.	Fir.	Decim. of a Bar.	Gallons	Decimals of a Bar.
1	.25			1	.25		
2	.5	18	.5	2	.5	18	.56250
3	.75	19	.5277777	3	.75	19	.59375
G.				G.			
1	.0277777r	20	.5555555	1	.03125	20	.62500
2	.0555555r	21	.5833333	2	.06250	21	.65625
3	.0833333r	22	.6111111	3	.09375	22	.68750
4	.1111111r	23	.6388888	4	.12500	23	.71875
5	.1388888r	24	.6666666	5	.15625	24	.75000
6	.1666666r	25	.6944444	6	.18750	25	.78125
7	.1944444r	26	.7222222	7	.21875	26	.81250
8	.2222222r	27	.75	8	.25000	27	.84375
9	.25	28	.7777777	9	.28125	28	.87500
10	.2777777	29	.8055555	10	.31250	29	.90625
11	.3055555	30	.8333333	11	.34375	30	.93750
12	.3333333	31	.8611111	12	.37500	31	96875
13	.3611111	32	.8888888	13	.40625		
14	.3888888	33	.9166666	14	.43750		
15	.4166666	34	.9444444	15	.46875		
16	.4444444	35	.9722222	16	.50000		
17	.4722222			17	.53125		

I. Malt or Corn Meaſure.		II. Malt or Corn Meaſure.		III. Malt or Corn Meaſure.	
Buſh.	Decimals of a Quar.	Gal.	Decimals of a Buſh.	Pints.	Decimals of a Gal.
1	.125	1	.125	1	.125
2	.25	2	.25	2	.25
3	.375	3	.375	3	.375
4	.5	4	.5	4	.5

Buſh.	Decimals of a Quar.	Gal.	Decimals of a Buſh.	Pints.	Decimals of a Gal.
5	.625	5	.625	5	.625
6	.75	6	.75	6	.75
7	.875	7	.875	7	.875

Note, *That Pints in Decimals of a Gallon of Liquid, are the ſame as theſe dry.*

17. *The Uſe of theſe Tables*, which is yet another Way of getting the Decimals of Beer, Ale or Malt-Meaſure. Suppoſe (as in the 3d Example under the 13th Rule above) it were required to know how 3 Firkins 8 Gallons 3 Pints are expreſs'd in the Decimal of a Barrel of Beer.

3 Firkins is .75 Bar.
8 Gallons is .2222, *&c.*
3 Pints is .01041625
Sum = .98263847 = the Decimal as before, but the 4 firſt Places will do.

But the 2 firſt are found in one Line, for 3 Firkins 8 Gal. are 35 Gal. Decimal .972r1.

Here you may obſerve, that tho' you have not in the Table of Beer-Meaſure, the Decimals of a Barrel that Pints are, you may very eaſily and ſpeedily find ſuch Decimal by multiplying the Decimal of 1 Gallon by the Pints given; as here .027r by 3, and divide the Product by 8, gives .01041625.

18. The Uſe of the Tables of Malt or Corn-Meaſure, to find what 5 Buſhels, 3 Pecks and 1 Gallon are in the Decimals of a Quarter. In 3 Pecks 1 Gallon are 7 Gallons.

And 5 Buſhels in the Decimals of a Quarter, by the firſt of the 3 laſt Tables is } .625 } Sum .7343 for Anſ.
And the 7 Gallons - - - - - - .1093

The latter of which Decimals is found by multiplying the Decimal of a Quarter that 1 Buſhel is, by 7, and dividing by 8, becauſe 7 Gallons is $\frac{7}{8}$ of a Buſhel.

Example 2. To find what Decimal of a Quarter 3 Buſhels, 2 Pecks, 1 Gallon, 5 Pints is, by help of the Table.

Firſt, conſider that your 2 Pecks 1 Gallon are 5 Gallons, and 5 Pints are 45 Pints, and there being 64 Pints in a Buſhel; therefore 45 Pints is $\frac{45}{64}$ of a Buſhel, and therefore take that Part of .125 (the Decimal that 1 Buſhel is of a Quarter) by multiplying by 45, and dividing by 64; thus,

.125
45
625
500
64) 5.62500 (.08789 = of a Quarter that 45 Pts. is.
505 (.375 = Decimal that 3 Buſh. is
570
580 .46289 = the Anſwer.
4

Or there being 512 Pints in 1 Quarter, the 45 Pints are $\frac{45}{512}$ of a Quar. = .08789, for as 512 to 45 :: ſo 100000 to .08789. And the Decim. of a Quarter that three Buſhels are, is found thus; 3 Buſhels is $\frac{3}{8}$ of a Quarter, and as 8 to 3 :: ſo .1000 to .375; which is agreeable to Rule 1.

and 15. of Reduction; by which alſo the Decimals in any of the 3 Tables laſt above are found.

For as 8 is to 1,2,3,4,5,6 or 7 :: ſo is 1000 to any Decimal in any of the 3 laſt Tables reſpectively: which ſhews their Calculation, as *per* Rule 15. foregoing. For the farther Uſe of thoſe Tables, ſee Rules 10, and 11, of Gen. Head II.

II. Having ſhewed many Ways of finding the Decimals of Money, Weight or Meaſure, I now come to ſhew how the Value of any Decimal may be found in the reſpective Parts of Coin, Weight or Meaſure. And

1. *Of finding the Value of Decimals* of Money, as ſuppoſe of .5432 of a Pound, anſwer, 10 *s.* 10 *d.* $1\frac{472}{1000}$ *qr.*

General Rule. Multiply by ſuch a Number of Units of the next lower Denomination, as make a Unit of the Denomination which your Decimal is of, and cut ſo many Places off toward the Right-Hand of the Product, as were in the Decimal given; and thoſe toward the Left-Hand of that Point or Daſh, are Integers of ſuch inferior Denominations; as in the Example, where the Value of .5432 is exhibited.

$$\begin{array}{rr} & .5432 \\ & 20 \\ \hline s. & 10.8640 \\ & 12 \\ \hline d. & 10.368 \\ & 4 \\ \hline qr. & 1.472 \end{array}$$

2. To find the Value of ſuch Decimals of a Pound as have any Digit in Primes Place, and a 5 or Cypher in the Seconds Place, (*i. e.* in the Places next, or next but one to the Point) divide ſuch Decimal by .05, and the Quote is the Value in Shillings.

Example of .25 *l.* of .40, of .75 and of .95, ſee under.

.05).25(5 *s.* Anſ. | .05) 40 (8 *s.* Anſ.
0 | 0
.05).75(15 *s.* Anſ. | .05).95(19 *s.* Anſ.
0 | 0

3. To find the Value of the Decimal of a Pound, by the Tables of Decimals of Money, under the 6th and 7th Rules of the 1ſt General-Head of Reduction. As ſuppoſe of .8762.

.8762
.85 = 17 *s.* ⎫ *s.* *d.*
.026 = 00 : 6 *d.* ⎬ Sum = 17 : $6\frac{1}{4}$: And ſo for
.0012 = 00 : 0 : 1 *q.* ⎭ others.

(1.) By the Table under the ſaid 6th Rule, againſt .85 (which is the next leſs than .87) ſtands 17 *s.* then of the 76 there remains 26. By the Table under Rule 7. the Value of the next leſs Decimal to .026 is .025, againſt which ſtands 6 *d.* Then the next Decimal to and leſs than .0012, in the ſaid Table, is .0010, againſt which ſtands 1 Farthing : So the Decimal of .8762 is valued 17 *s.* $6\frac{1}{4}$.

4. *A 4th Way of valuing the Decimals of a Pound*, may be truly ſaid to be by Inſpection; for by having the Rule perfect, any one may (as I have often done) expreſs the Shillings, Pence and Farthings, which are the Value o Decimals, as faſt as if thoſe Shillings, &c. were actually wrote down.

The Rule is, double the Place next the Poin (or Points) is ſo many Shillings, and if the SecondsPlace be 5 or more, add 1 *s*. Thirdly, what the Seconds Place are over or under 5, call them ſo many Tens, and the Thirds Place ſo many Units of Farthings. But if thoſe Farthings exceed 12 upward to 38 excluſive, make them leſs by 1; and if 38 or upward, deduct 2; and the remaining Farthings are Pence and Farthings, ſo many as they contain.

Examples.

.876 of a Pound Sterling is 16 *s*. more 1 *s*. more 26 Farthings; leſs by 1 is 25 or 6 *d*. $\frac{1}{4}$ So the Anſwer is 17 *s*. 6 *d*. $\frac{1}{4}$ as above.

.943 is 18 *s*. and 43 *qr*. leſs by 2 is 10 *d*. $\frac{1}{4}$. Anſwer 18 *s*. 10 *d*. $\frac{1}{4}$

.8635 is 17 *s*. and 12 *qr*. or 17 *s*. 3 *d*.

In this laſt Example, I make the 13 *qr*. leſs by 1, and the reſt with the 5 in the 4th Place, are juſt 3 *d*. as by the Table, *viz*. .0125, for twice, that is .025 or juſt 6 *d*. being made leſs by 1. For the Reaſon why we make the Farthings leſs by 1 in 25, is becauſe the Rule ſuppoſes 1000 Farthings in a Pound, and there being but 960, therefore 1000 is 40 too many, 500 is 20, 250 is ten, and conſequently 25 is 1 too many; and 13, &c. I make leſs by 1, becauſe it is above half of 25, and is therefore above half a Farthing too much, if taken for 13 Farthings. And thus I have given you a ſhort Rule, and the Foundation or Reaſon of the moſt abſtruſe Part of it, (never done by any other Author,) for as to the other Part of doubling Primes Place, it is becauſe whatever Digit ſtands next the Point, is ſo many 2 *s*. for .1 is 2 *s*. and conſequently .2 muſt be 4 *s*. .4 8 *s*. &c. and this you may prove by the 1ſt Rule in Reduction, and alſo that 5 in Seconds Place is 1 *s.* for it is Half of .1 or of 2 *s.*

5. To reduce Decimals of Averdupois Weight to their Values in lower Denominations, as .7654 of 1 *C.* or of 112 ℔.

Work as *per* General Rule under the 1ſt Rule above, of this 2d Part of Reduction.

.7654 C.
4 Quarters of C. } multiply
qrs. 3.0616
28 } multiply by *l.* in a *qr.*
℔. 1.7248
16 Oz. in a Pound } multiply
oz. 11.5968
16 Drams in an Oz. } multiply
dra. 9.5488

q. l. oz. dr.
So the Anſ. is = 3 : 1 : 11 : 2 $\frac{5968}{10000}$

I ſhall ſhew how to multiply by any 2 Figures (as is done by thoſe above) when I come to Multiplication, by only making one Product or Line.

6. To find the Value of Decimals of Beer-Meaſure, as of .7654 of a Barrel.

Work as per the General-Rule, as in the Margin.

Bar. .7654
4 Firk } multiply
Fir. = 3.0616
9 Gal. in a Firk.
Gal. 0.5544
8 Pints in a Gal.
Pts. = 4.4352

F. G. P.
Anſw. 3 : 0 : 4 and near ½

7. To find the Value by another and ſhorter Way, as of .4567 for the Value in Gallons. Rule, Multiply by 4, and the Product is 18268; ſubſtract each Digit of that from its Right Hand Digit, and the Remainder is 16.4412 Gallons. As thus, 8 from 10 (ſuppoſing a Cypher toward the Right-Hand) reſt 2, 1 and 6 is 7 from 8 reſts 1, 2 from 6 reſts 4, 8 from 2 or 12 reſts 4, 1 and 1 is 2 from 8 reſts 6, and 0 from the 1 reſts 1; ſo the Remainder is 16.4412, cutting off as many for Decimals as in the Decimal given.

So .1234 of a Barrel is 4936, or Gallons 4.4424
.234 — — — 939, or — — 8.424
.3456 — — —13824, or — —12.4416
.4567 — — —18868, or — —16.4412
.89 — — — 356, or Gal.—32.04

Here you may ſee that inſtead of 2 Multiplications (as by 36) and an Addition, you do the Work by 1 Multiplication only by 4, and an eaſy Subſtraction by this Way of my own Contrivance. To find the Value of the Decimal of a Gallon, is ſhew'd under Rule 6 above.

8. To find the *Value* of the Decimal *of Malt or Corn-Meaſure*; as of .3456 of a Quarter, in Buſhels, Pecks, Gallons and Pints.

.3456 Quarters } multiply
8 Buſhels in a Qr. }

Buſhels = 2.7648 Parts of a Buſh. } multiply
4 Pecks in a Buſh. }

Pecks = 3.0592 Parts of a Peck } multiply
2 Gallons }

.1184 Parts of a Gal.
8 Pints in a Gal.

.9472 Parts of a Pint } multiply
4 Quar. of a Pint }

Qr. Pts. = 3.7888

buſh. pec. gal. pts. q.pts.
So the Anſwer is = 2 : 3 : 0 : 0 : 3.79 ferè.

Example 2. To find the Value of .6543 of a Quarter.

.6543
8 *buſh. peck. gal. pts.*
Anſw. 5 : 0 : 1 : 7

5.2344
4

.9376
2

1.8752
8

7.0016

9. To find the Value of the Decimals of a Foot in Inches and Quarters, proceed as by the General-Rule foregoing, and as follows.

.8765 of a Foot } multiply
12 Inches }
———
Inches = 10.5180
4 Quar. of an Inch } multi.
———
Qu. Inch. = 2.072

10. But I have contriv'd this Way following, to find the Value by Inſpection. If the Primes Place (or that next the Point) be 5 or upward, call it 6 Inches; then for what is over or under 5, call them ſo many Inches and ſo many Quarters: And for thoſe in Seconds Place, call them ſo many Half-Quarters, omitting the odd Unit, if there be one. So in the Example above, I put 6 Inches, and the 8 being three above 5, I call them 3 Inches and 3 Quarters, which make 9 *Inch.* 3 *qr.* And half of 7 in the Seconds Place, is 3 (omitting the odd one) Quarters, and 9 *Inch.* 3 *qr.* make 10 *Inch.* 2 *qr.* as by the Work. Remember always to deduct 1 from the Figure in Seconds Place.

A Table of Inches and Quarters of Inches in the Decimals of a Foot, follows.

Inches :	*Qr.*	*Decimals of a Foot*
	1	.02083
	2	.0416r1
	3	.0625

Inches		*Decimals of a Foot.*
1 :	——	.083*r*1
2 :	——	.16*r*1
3 :	——	.25
4 :	——	.3*r*
5 :	——	.416*r*1
6 :	——	.5
7 :	——	.583*r*1
8 :	——	.6*r*
9 :	——	.75
10 :	——	.83*r*1
11 :	——	.916*r*1

By this Table you have not only the Decimals of any Number of Inches and Quarters in thoſe of a Foot, but alſo the Value of any Decimal of a Foot; for Inſtance, in the Example above of finding the Value of .8765 of a Foot.

The next leſs Decimal in the Table is .83*r*1, againſt which is which Decimal 83*r*1 taken from that given, there reſts .04317; againſt the next leſs than which, ſtands

So the Sum is 10 *In.* 2 *qr.* as before.

	Inch.	*Qr.*
——	10 :	0
——	0 :	2
Sum =	10 :	2

11. In like manner, to find the Value of any Decimal of a Barrel of Beer, by the Decimal Table under the 16th Rule of the 1ſt Head, above: As ſuppoſe, of .9826, the next Decimal leſs than that, in the Table, is .972*r*1, againſt which ſtands 35 Gallons, or 3 Firkins 8 Gallons; and the Difference of the 2 Decimals is .01037 of a Barrel, which multiply'd by 36, and that Product by 8, gives 3 Pints ferè: But where Pints are in the Value, the beſt way to find the Value, is that under the 6th Rule above.

12. To find the Value of the Decimal of a Quarter of Malt, *&c.* by the Tables under the 16th Rule of the 1ſt Head, above; as of .6234. Look in the Table for the next leſs, and againſt it you will find 5 Buſhels the Value.

Example 2. To find
the Value of ——— — .4629 of a Quarter;
look on the Table and
you find ——— —— .375 the next leſs = 3 Bu.
.0879 Difference.
which multiplied by 8,
gives ——— ——— .7032 of a Buſhel.
Look in the Table for
the next leſs, is ——— .625 againſt which is 5 Gal.
.0782 Difference,
which multiplied by 8,
gives ——— ——— .6256 of a Gallon:
The next leſs than which
in the Table ——— .625 and againſt that
is = 5 Pints.

Anſwer, 3 *Buſh.* 5 *Gal.* and 5 *Pints*; which proves the 2d Example under the 18th Rule of the 1ſt Head of this Section.

Sect. 3. *Addition of Decimals.*

1. *Rule.* WHether your Numbers given be all Decimals or mixt with whole Numbers, add them as Integers; and in order rightly to do that, you muſt place Point under Point, and then add thoſe of the ſame Place together.

Exam-

Example 1.	*Example* 2.	*Example* 3.
	.9	
.098765	98.76	1123.45678
.4321	5.4321	990.1234
.9876	0.987	56.987
.543	654.3219	7.89
.219876	87.65	65.4
.543	4.321	789.345
.21	.09	6789.1234
	.8	56789.12345
Sums=3.034341	853.262	66611.44903

2. When Numbers are ſuch that the Digets in any Line are 1, 2 or 3, *&c.* and the ſame, but are ſuppoſed to be repeated infinitely, and are accordingly marked with *r* or *r*1, *r*2, *&c.* as under Rule 13. of Reduction, General-Head I. Then you muſt conſider what the Sum of thoſe repeating Figures would be, if wrote down, and carry ſo many Units to thoſe that are expreſs'd in writing: As in the Examples.

Exam. 1.	*Exam.* 2.	*Exam.* 3.
.9*r*	.98*r*2	
.8*r*	.76*r*2	.987*r*3
.7*r*	.54*r*2	.654*r*3
.6*r*	.32*r*2	.321*r*3
.5*r*	.10*r*2	
3.8*r*=Su.	=2.7*r*2 Su.	=1.963*r*3

3. But becauſe it more rarely happeneth that all are repeating Numbers, in a Sum to be added; and that ſuch Numbers are, for the moſt part, mixt with others; take an Example or two of ſuch.

Exam.

Example 1.	*Example* 2.	*Example* 3.	
.1234*r*1	6.7*r*	1234.5678	
.5678	89.123	912.34*r*1	
.912	34.567	56.78	
.34	788.9*r*	9.8*r*	
.5*r*	345.6	.9*r*	
2.4988	1265.067*r*1	2214.58113*r*1	Sums

The repeating Digits being ſuppoſed (as in the 3d Example) to come farther to the Right, by 2 Places, than the 8, I add them, ſaying, 9 and 8 is 17 and 4 is 21, which 2 carried, makes 23, and the 3 doth repeat: Then ſay, 2 carried and 9 is 11, and 8 is 19, and 4 is 23, and 8 is 31, put down 1, and ſo proceed: 3 carried and 9 is 12, and 8 is 20, and 4 is 24, and 7 is 31; I put down another 1 and carry 3, and ſay, 3 and 9 is 12, and 8 is 20, and 8 is 28, and 4 is 32, and 6 is 38; put down 8 and carry 3 to Primes Place of Decimals, &c.

4. There are 2 Ways which demonſtrate the Truth of adding Decimals.

(1.) Either by adding their Values; or,

(2.) By adding Decimals as Vulgar Fractions: As to the 1ſt,

	l.		*l.*	*s.*	*d.*
Add	.75	———	— :	15 :	—
	.125	———	— :	2 :	6
	.5	———	— :	10 :	—
	.25	———	— :	5 :	—
	.625	———	— :	12 :	6
	2.25	Sum =	2 :	5 :	—

and ſo are the Decimals.

2. And admit .5 be added to .625, or $\frac{5}{10}$ to $\frac{625}{1000}$, the Fractions in a common Denominator are $\frac{5000}{10000}$ and $\frac{6250}{10000}$; and the Sum of the Numerators is 11250, which divided by the Denominator, is 1.125; and that is the Sum of the Decimals given, added the Way as taught above, which is 1 *l.* 2 *s.* 6 *d.*

Sect. 4. *Subſtraction of Decimals.*

1. YOu may, with ſome Things to be obſerved, ſubſtract Decimals as Integers: For it ſometimes happeneth, that the Number you deduct from, tho' greater in Value, yet conſiſts of fewer Places; in which Caſe, you muſt imagine Cyphers towards the Right-Hand of the Greater, and borrow 10 as common.

2. You muſt place the Decimals given as you are directed in Addition, Point under Point.

	Example 1.	*Example* 2.	
From	.987	3.45	123.0
Deduct	.1234	.9765	.9876
Remains	.8636	2.4735	122.0124

3. As to repeating Digits, you muſt, as in Addition is taught, work as tho' they were actually wrote down.

	Example 1.	*Example* 2.	*Example* 3.
From	98.5	4.9*r*	1.234*r*3
Take	9.5*r*	3.89*r*2	.56*r*2
Reſts =	88.94*r*1	1.10*r*2	0.668577*r*6

Example 4. A Copper 36 Inches deep, holds in the Whole of it 32 Barrels 1 Firkin 5.3*r* Gallons; and it being gauged, is found to contain in the Middle of every 6 Inches of the Depth to the Crown, as follows.

	Bar.	*Fir.*	*Gal.*	
At 3 Inches from the Top	5	3	6.7	
9 —— ——	5	2	1.2	
15 —— ——	5	—	5.1	
21 —— ——	4	3	—6	
27 —— ——	4	1	5.2	
33 —— ——	4	—	1.7	
				(to the Crown.
	29	3	2.5	Sum, Content
	32	1	5.3*r*	Content of all
	2	2	2.83*r*1	= the Anſwer, reſts.

What is the Quantity required to cover the Crown? This is found by taking the Sum of the Contents above, and Subſtracting that from the whole Content: See the Work.

4. The Rules for ſubſtracting Decimals, may be proved true from the two Ways mentioned in Addition: For if

		B.	*F.*	*G.*	*Pts.*
From	11.625 Bar. =	11	2	4	4
I take	9.75 —— =	9	3	—	—
Remainder will be	1.875 equal to	1	3	4	4

(by the 6 Rule of 2d Head of Reduct.

And for the ſecond Way of Proof, which alſo proves the Rule of Subſtraction: For if from $11\frac{625}{1000}$ I take $1\frac{875}{1000}$, the Remainder will be $9\frac{75}{100}$; for the Decimals having a common Denominator $\frac{875}{1000}$ taken from $11\frac{625}{1000}$, there remains $10\frac{750}{1000}$, from which taking the 1, and cutting off the Cyphers towards the Right-Hand, the reſt will be $= 9.75$, as in the laſt Example it is.

$$\begin{array}{r} 11.625 \\ 875 \\ \hline 10.750 \end{array}$$

Sect. 5. *Multiplication of Decimals.*

I Have elſewhere ſhew'd, that this Rule diſpatcheth the Work of many Additions, in a wonderful compendious Manner. For if it was required to know how many Inches ſolid there are in 40 Gallons of Ale or Beer; there being 282 Inches in one Gallon, the Way to do this without Multiplication, muſt be to put down 282 forty times, and add them together; but Multiplication does the Work with a thouſandth Part of the Trouble and Time. Thus,

$$\begin{array}{rl} 282 & \text{ſolid Inches in a Gallon.} \\ 40 & \text{Gallons} \\ \hline 11280 & \text{Inches Anſw.} \end{array}$$

This Rule of Multiplication is demonſtrated by *Euclid* 1, 2, and 3 of the 2d Book.

1. There is no Occaſion here for a Multiplication Table, becauſe it is ſuppoſed that they

who would learn Decimals, are not unacquainted with Vulgar Arithmetick; the chief Thing therefore to be added, as to Multiplication of Decimals, is only *This General Rule.*

Cut off from the Product, by a Point, ſo many Places toward the Right-Hand of your Product for Decimals, as there are of Decimal Places in the Factors, (or Multiplicand and Multiplier.) And

Iſ there be not ſo many Places, put 1 *or more Cyphers towards the Left-Hand, next the Point, as will make up that Number.* It matters not how the Factors are placed, but I do it as in Integers.

Example 1.	*Example* 2.	*Example* 3.
17.16	123456	45.6789
19.5	.00123	.000456
8580	370368	2740734
15444	246912	2283945
1716	123456	1827156
334.620	151.85088	.0208295784

To apply the General-Rule above, it is plain that in the 3 Examples I cut off from the Products, as many for Decimals, as there are Decimal Places in the Factors: And in the 3d Example, becauſe there wants 1 of ſo many Places, therefore I make up the Number 10, by putting a Cypher toward the Left-Hand, and next the Point in the Product.

2. *A Demonſtration that any Number multiplied by a Fraction, gives a Product leſs than the Multiplicand.*

In the 3d Example you ſee that the Multiplicand is above 45, but the Product is little more than 2 hundred Parts of Unity: So that *multiplying by a Fraction*, is ſo far from increaſing the Multiplicand, that it always gives a Product leſs than it.

2. And in this Caſe, the Product is always ſuch a Part of the Multiplicand, as the Multiplier is of Unity.

This will appear more plain in a leſſer and even Number, as if you multiply *16* by $\frac{1}{2}$ or .5, the Product will be 8: So that as the Product 8 is Half of *16*, ſo is the Multiplier .5 Half of Unity.

Or if you multiply *16* by $\frac{1}{4}$ or .25; the Product will be 4, which is a 4th of *16*: Whence ariſeth this

Corollary 1.

That the Product and Quotient will be the ſame, if you multiply any Number by any Decimal Fraction, as if you divide that Number by the Denominator of a Vulgar Fraction, (whoſe Numerator is 1, and) which is equal to that Decimal. Thus if you multiply *16* by .5, the Product will be 8, equal to the Quote: If *16* be divided by 2, the Denominator of $\frac{1}{2}$ equal to .5; and *16* multiply'd by .25 produceth 4, equal to *16* divided by the Denominator of $\frac{1}{4}$ equal to .25.

Corol-

Corollary 2.

And if the Vulgar Fraction, equal to the Decimal multiplying, have a Numerator above 1, then the Product will be ſo many times more than if the Numerator were but one.

Thus to multiply 16 by $\frac{3}{4}$ (or .75) the Product will be 12, which is $\frac{3}{4}$ of 16, as $\frac{3}{4}$ is 3 Quarters of one, and 12 is three times the ſaid 4.

Or in Decimals themſelves: If 16 be divided by 100, it is .16; 75 of which Parts is 12, as before: And in the ſaid 3d Example above, the Product is $\frac{456}{1000000}$ of the Multiplicand, which if you divide into a Million of Parts (as the Multiplier is) 'twill be .0000456789, which multiplied by 456, will give the ſame Product as before, and ſhews that $\frac{208295784}{10000000000}$ of a Unit, is equal to .0000456789 multiplied by 456, or 45.6789 multiplied by .000456.

And indeed nothing can be more evident than that if multiplying any Number by *one*, produce a Number equal to the Multiplicand, it muſt as certainly follow, that multiplying by leſs than one, will produce leſs than the Multiplicand; and that multiplying by more than one, will produce more than the Multiplicand.

	Example 4.	*Example* 5.	*Example* 6.
Multiply	.34500	=.125	=.0625
by	.980	8	80
	.3381	1.000	5.0000

3. If Cyphers be at the End of one or both Factors, you need take no farther Notice of them, but to place them at the End of the Product; but if at the End of Decimals, they may be wholly omitted, as in Example 4.

4. The 5th and 6th Examples ſhew that the Demonſtration above, holds where the Multiplicand is a Fraction: as in the 5th, I multiply $\frac{1}{8}$ by 8, and it produceth 1; and in the 6th Example, $\frac{5}{80}$ by 80, produceth 5.

There are four other Things to be treated on, before I leave *Multiplication*, in order to make this Section a compleat Diſcourſe on that Part of Arithmetick: As,

5. *Shew how to multiply and have only ſo many decimal Places as you think beſt, avoiding all ſuperfluous Figures in the Product.*

Example 1. To multiply 17.16 by 19.5, and to have only 2 Decimal Places in the Product.

Rule. Invert the Places of your Multiplier, and put Units Place of the integral Part, under that Part or Place of the Multiplicand, as anſwers to the Number of Places which you propoſe to be in the Product.

17.16
5.91

1716
15444
858

Prod. = 334.62

Then multiply the ſeveral Figures of the Multiplyer into the Figures ſtanding over it, and into the reſt toward the Left-Hand, minding, however, what would be carried from the Product of the Figure towards the Right-Hand of that where you begin to multiply. The

Examples will render the Rule very perſpicuous and clear ; where,

In the Example you ſee that Units are placed under Units of the ſeveral Products, and not as in the common Way.

And becauſe there is nothing over the firſt Figure, I place a Point under that, for under that, the reſt of the firſt Figures in the Products muſt be placed ; and you ſee the Product is the ſame as in the 1ſt Example under the 1ſt Rule of this Sect. 5.

Example 2. multiply 123456 by .00123, and to have 5 Decimal Places in the Product.

According to the Rule, I add ſo many Cyphers, till I can place that which would be Units Place of the Multiplier (which I mark with a Point) under the 5th Decimal Place, and then I multiply. Place the Products, and add as you ſee in the Example, which is proved to be right by the 2d Example of Multiplication.

```
123456.00000
      32100.
------------
12345600
 2469120
  370368
---------
151.85088 = Prod.
```

Example 3. Of this 5th Rule, where we will only have 2 Decimal Places in the Product (as being ſufficient, if Money, and the Product not to be multiplied by any Thing) of the laſt Example, or Example 2. of this Section. The chief

```
123456.00
  32100.
---------
12346
 2469
  370
------
151.85 Product
```

Thing to be conſider'd, being to place aright the Units Place of the Multiplier, (which ſuppoſing there were one,) muſt, as here, be under the 2d Cypher, becauſe only 2 Decimal Places are to be in the Product; and work as in the Operation before you, and you have the true Product required.

6. The ſecond Thing I propoſe to compleat Multiplication, is, To ſhew ſome brief Ways to perform the ſame: As in theſe Examples.

	Examp. 1.	*Examp.* 2.	*Examp.* 3.
Multiply	234.56	23.456	2.3456
By	13	15	19
Product=	3049.28	351.840	44.5664

Here you ſee 'tis not neceſſary to make 2 Lines in multiplying by any Digit in Units, and 1 in Tens Place: For I ſay, in Example 1. 3 times 6 is 18, 8 and go 1; three times 5 is 15, and 1 is 16, and 6 in Units Place is 22, 2 and carry 2; 3 times 4 is 12, and 2 is 14, and 5 (in Tens Place of Multiplicand) is 19, 9 and go 1, *&c.* adding the Figure in the Place next the Right-Hand, to that which you laſt multiplied.

7. And ſo you may do by adding 2 Places, if there were 11 before the 3, 5, 9, *&c.* as in theſe Examples.

Exam. 1. 23456.	*Exam.* 2. 23456	*Exam.* 3. 23456
11.3	1.15	.119
265052.8	2697440	2791.264

8. To multiply by any 2 Figures under 100, and make but 1 Line of Product.

	Example 1.	*Example* 2.	*Example* 3.
	3456.7	345.67	34.567
	34	5.6	.78
Prod.	=117527.8	=1935.752	=26.96226

In theſe Examples, for inſtance in the 1ſt, I ſay 4 times 7 is 28, 8 and go 2; 4 times 6 is 24, and 2 is 26, and 3 times 7=21, is 47, 7 and go 4; 4 times 5 is 20, and 4 is 24, and 3 times 6 = 18, is 42, 2 and go 4; 4 times 4 is 16, and 4 is 20, and 3 times 5=15, is 35, 5 and go 3; 4 times 3 is 12, and 3 is 15, and 3 times 4=12 is 27, 7 and go 2, and 3 times 3 is 9, and 2 is 11 :. And in like manner all others.

What might here properly come in, as to Cyphers toward the Right-Hand of 1 or both Factors, you have it under the 3d Rule foregoing: But I ſhall inſert this Multiplication-Table, becauſe 'tis a Curioſity of my own Invention.

A new and brief Multiplication Table.

Two of theſe multiply.	1	9	40	Two of theſe add, reſpecting the given Numbers in the Middle.
	2	8	30	
	3	7	20	
	4	6	10	
	5	5	0	

As 8 times 6 is 30 and 10, and 2 times 4 =48.

9. There is an eaſy and brief Way of caſting up the Value of any Thing, when the Value of the Integers is Pounds, Shillings, Pence, *&c.* without reducing the Value into the loweſt, and

afterward the whole into the higheſt Denomination: As to caſt up the Value of 72 Barrels of Beer at — 2 : 12 : 3 each

Here, becauſe I can divide the 72 by 9, or 8, without a Remainder, I therefore multiply by 9 and that, Product by 8. (9 times 8 making 72.) But if it were 76 Barrels, multiply the given Number by 4 more, and add the Product as here.

	l.	*s.*	*d.*	
	2	12	3	each
			9	
	23	10	3	
			8	
l.	188	2	—	Anſ. at 72 (Bar.
	10	09	—	
l.	198	11	—	Anſ. at 76 (Barrels.

10. The third Thing I ſhall treat on, to make this Section of Multiplication full and entire (which by ſome Authors is taught by only a ſmall Part) is

To ſhew the Management of repeating Digits; which is new and my own Thought.

1. A whole or mixt Number by a Decimal, as 34.567 by .6r.

34.567
.66666r1
207402
23.0446r1=*Prod.*

Here Note, That you always cut off 1 more than the Decimal Places that are in the Multiplicand.

In this Caſe I only multiply by one 6, and add the Product from Right to Left; thus, 2 and 4 is 6, and 7 is 13, and 2 is 15, which 5 and 1 is 6 = the repeating Digit of the Product: Then I proceed with the 1 of the 15, and ſay, 1 and 4 is 5, and 7 is 12, and 2 is 14; put down 4, and ſay 1 and 4 is 5, and 7 is 12, and 2 is 14; put 4 down and carry 1, and 7 is 8, and 2 is 10; put down 0, and ſay 1 and 2 is 3, 0 and 2 is 2: So the Product is 23.0446*r*1.

Note, This is prov'd by dividing the Product by .66666 or ſo many Sixes as you would have Places in the Quotient.

Example 2.

Mult. 1234

by 33.3*r*

3702

41133.3*r* = *Prod.*

This you may prove by multiplying by the Decimal, as before, and adding the Product to that of 1234 by 33.

In this 2d Example you work as in the laſt, and put as many of the repeating Figures as are in the integral Part of the Multiplier toward the Left-Hand of the Point in the Product: As 33 I put between 411 and .3 and conſider what would be carried in multiplying the Repeaters in all Examples.

Example 3.

Mult. ———3.55555 *&c.*

by ———4.66666, *&c.*

or 3.5*r*1

by 4.6*r*1

213

2.36*r*1

14.2*r*1

16.58*r*1 = *Prod.*

Note, *r ſtands for repeating.*

11. *The laſt Thing to compleat this Section, is to ſhew ſeveral Ways of proving Multiplication.*

1. By Diviſion; by which we will prove the 1ſt Example of Multiplication, by dividing the Product by the Multiplicand, and the Quotient will be your former Multiplier.

```
17.16) 334.620 (19.5 = the Multiplier
       16302
         8580
            0
```

2. Or it may be proved by dividing the Product by the Multiplier, and the Quote is the Multiplicand.

```
19.5) 334.620 (17.16 =the Multiplicand.
      1396
        312
       1170
          0
```

3. It may be proved by multiplying the Multiplicand by any 2 or more Parts of the Multiplier, adding the Products together; as by 19, and then by .5. Thus,

```
        17.16       17.16
           19          .5
        -----       -----
Add { 326.04        8.580
    {   8.58
      ------
      334.62 = the Product as before.
```

4. By multiplying the Multiplier by any Parts of the Multiplicand, that make up the Whole ; as 19.5 by 17 and by .16.

Or, 5. The Multiplier by the whole Multiplicand.

19.5	19.5
17	.16
331.5	3.120
3.12	
334.62 = the Product of the (1ſt Example.	

And many other Ways might be ſhew'd, but theſe may ſuffice, being moſtly new ; and that Way by caſting out the Nines, I have elſewhere prov'd falſe.

Sect. 6. *Diviſion of Decimals.*

THERE are 9 Caſes in Diviſion, of each whereof I ſhall give an Example, without varying the Digits. It being pre-ſuppoſed that our Reader underſtands Diviſion of Integers, and what follows will fully inſtruct him how to manage the Decimals that can occur in Diviſion, by this ſhort

General Rule. Firſt, Take care that there be ſo many Decimal Places in the Dividend, as the Sum of thoſe is, which you have in the Diviſor, and would have in the Quotient ; and conſequently you muſt always point off toward the Right-Hand of your Quotient, ſo many Places as the Difference is between the Decimal Places in the Dividend and Diviſor.

Example 1. Divide 151.85088 by 123.

```
123) 151.85088 (1.23456 = Anſwer.
       .  .....
      288
      425
       560
        688
         738
           0
```

Here you ſee that I point off 5 for Decimals in the Quotient, according to the latter Part of the Rule above.

2dly, If the Diviſor were a mixt Number, as 1.23, you muſt divide as before, and cut off the Difference between 2 and 5 in the Quote, which will then be 123.456.

3dly, If both were Decimals, as .15185088 by .123, the Quote has the ſame Digits, but has the Difference between 8 and 3 cut off by the Point, which is the ſame as in the firſt Example.

4thly, If the Diviſor be very ſmall, as .00123, and the Dividend as in the 1ſt = 151.85088, then the Quote will be 123456, there being no Decimals, becauſe no Difference between the Decimal Places in the Parts given.

5thly, If the Dividend be a very ſmall Decimal, and the Diviſor a whole Number, as .015185088 by 123. In this Caſe the Manner of Operation will be, as under the firſt Example: But the Difference of Decimal Places in the Numbers given, being 9, and there being but 6 in the Quotient; you muſt therefore make up 9, by adding 3 Cyphers next the Point, ſo the Anſwer will be .000123456.

6thly, If you divide a whole Number, as 15185088 by a mixt, as 1.23, you muſt add, at leaſt, ſo many Cyphers to the Right-Hand of the Dividend, as there are Decimal Places in the Diviſor, and as many more as you would have Decimal in the Quotient, according to the General Rule above, and then the Quotient will have ſo many Places more of Integers, as there were Cyphers added to the Dividend: ſo 2 Cyphers added, brings out 2 more in the Quote; all which, and the 7th Caſe (if dividing a ſmaller whole Number by a greater) and the 8th of a whole Number by a Decimal; and the 9th of a Decimal, by a mixt Number, are very eaſily perform'd by obſerving the General Rule aforeſaid.

Exam.	*Diviſors.*	*Dividends.*	*Quotients.*
1	123	151.85088	1.23456
2	1.23	151.85088	123.456
3	.123	.15185088	1.23456
4	.00123	151.85088	123456
5	123	.015185088	.000123456
6	1.23	15185088.00	12345600
7	123	15185088	123456
8	.0123	15185088.0000	1234560000
9	12.3	.15185088	.0123456

2. The Truth of any of theſe Examples may be proved by only multiplying any Quote by the reſpective Diviſor, and the Product will be the Dividend which ſtands between them: So in the 8th Example, if you multiply 1234560000 by .0123, the Product will be 15185088, *&c.*

2dly, A ſecond Way to prove Diviſion, is to divide the Dividend by the Quote, and that Quotient will be the former Diviſor.

3dly, Divide by double the Diviſor, and twice the Quote is the former Quote. As if you divide .15185088 by 24.6 (in the 9th Example) the Quote will be .0061728, which doubled is .0123456, as above. Or,

4thly, If the Units Place of your Diviſor admits of it, take any Part and divide the Quote by the Part you took, and that Quotient is your former Quote. So in the ſaid 9th Example I having taken a 3d of my Diviſor, 'tis 4.1, by which dividing the .15185088, the Quotient is .0370368, a 3d of which is .0123456, the Quotient in the Example above.

A 5th Way (where the long Way of Diviſion is uſed) is to add the ſeveral Products together, will make your Dividend, with the Remainder, if any be. Example.

```
12) 20738 (1728
    . . . .
    12 ——— 12
    ——
     87
     84 ——— 84
     ——
      33
      24 ——— 24
      ——
       98
       96 ——— 96
       ——
        2 ——— 2
    Sum = 20738
```

3. Some Uſes of Diviſion: As ſuppoſe 76 Barrels of Beer coſt *l.* 198 : 11, or 198.55, what is that *per* Barrel?

l. *s.* *d.*
76)198.5500(2.6125 Anſw. or 2 : 12 : 3
465
95
190
380
0

4. This may alſo be done without Decimals, as by reducing the 198 *l.* 11 *s.* into Shillings, and dividing by 76, *&c.*

5. Or without reducing the given Sum, it may be done thus.

l. *s.* *d.*
76)198:11(2 : 12 : 3 = Anſwer.
46
931 *s.*
171
19
228 *d.*
0 reſts.

6. If 35 Quarters of Malt coſt 44 *l.* 13 *s.* 9 *d.* what will 1 Buſhel coſt? See the Work decimally.

35)44.688(*l.* 1.276 = 1 Quarter.
96 $\frac{1}{8}$ of which is = .1595.
268
238 Or 3 *s.* 2 *d.* $\frac{1}{4}$ *per* Buſhel.
28

7. Or this may be done by taking a 5th of a 7th of the given Number for the Price of the Quarter, and $\frac{1}{8}$ of that for the Price of the Buſhel: Thus,

l. *s.* *d.*
44 : 13 : 9
$\frac{1}{7}$ = 6 : 7 : 8
$\frac{1}{5}$ of that = 1 : 5 : 6 $\frac{1}{4}$ the Quarter, or
(3*s.* 2 *d.* $\frac{1}{4}$ theBuſh.

Example 2. Decimally what is the Price of the Buſhel, when 65 Quarters and 5 Buſhels coſt 89 *l.* 17 *s.* 6 *d.* or *l.* 89.875

65.625) 89.875000 (1.369 *l.* Anſw. the Quarter
242500 ($\frac{1}{8}$ = .171 *l.* the Buſhel.
456250
625000
34375 Or 3 *s.* 5 *d.* ferè.

Thus much is ſufficient for the Doctrine of Decimals, which upon Compariſon with what has been taught by moſt Authors, will be found moſt copious, eaſy and applicable to the Purpoſe we are upon, and in many Things new, and altogether pertinent and uſeful.

CHAP. II.

The Extraction of Roots.

Sect. 1. *Of the Square Root.*

THE Square Root of any Superficies is a Line, which being multiply'd in it ſelf, produceth the Superficies given: So that by this Extraction, any kind of Figure, as Circle, Ellipſis, Triangle, *&c.* is reduced to a Geometrical Square, and muſt therefore neceſſarily be of great Uſe in Menſuration and Gauging.

2. The Square of any Number is the Product of that Number multiply'd in it ſelf. A Table of ſingle Roots (or thoſe conſiſting of one Place) and their Squares, follows.

Roots	*Squares*	*Roots*	*Squares*
1	1	6	36
2	4	7	49
3	9	8	64
4	16	9	81
5	25		

3. What is the Side of a Square Figure equal to one whoſe Area is 144 Inches, or 1 Square Foot.

Here you ſee you muſt firſt point over every other Figure, beginning with Units Place, which Points ſhew you the Number of Places that will be in the Root.

$\dot{1}4\dot{4}$(12 = Root.
2) $\dot{4}4$
0

4. Conſider what Number being ſquared will have that Square next to and leſs than the firſt Branch or Point, which is here 1.

5. Which 1 I therefore put in the Root, and ſubſtract the Square of it from the 1 pointed under, and the Remainder is 0; to which,

6. I bring down the next Branch 44.

7. I double the 1 put in the Root for a Diviſor to the 44, *&c.* except Units Place (which muſt always be reſerved to deduct the Square of the Quote, or Figure now to be put in the Root from) then,

8. I aſk how often 2 in 4? the Anſwer is 2; which I put in the Root, and deduct the Square thereof from Units Place of the 44, and there reſts 0: Then I ſay, 2 times 2, the Diviſor, from the other 4, and there reſts 0. So the Root of 144 is found = 12.

Example 2. To find the Square Root of 12345678.

```
        .  .  . .
        12345678(3513.64
     6) 334
    70) 956
   702) 25578
  7026)  4509.00
 70272)   29304.00
 ————————— 119504reſts.
```

You may, by this Example, ſee that there is little more than a Repetition of the former Rules in the laſt Example.

For having pointed over every other, I conſider what Number being ſquared will produce a Number next to and leſs than the firſt Branch 12, which is 3; ſo I put 3 in the Root, and taking its Square from 12, there reſts 3; to which I bring down the 2d Branch 34, which divide by double the 3 laſt put in the Root, and the Quote is 5: Then I ſquare 5, is 25 from 34 (4 in Units Place of 334, and 3 borrow'd) and there remains 9; 5 times 6 the Diviſor, is 30, and 3 borrowed is 33, from 33 reſts 0: So to the 9 I bring down the next Branch 56, and then double 35 in the

Root, makes 70 for a new Diviſor, whereby to divide 95 of the 956, and ſo proceed as taught before, and in the Example, till you come to have the Remainder 4509; to which

Bring or put 2 Cyphers toward the Right, and ſo proceeding as before, you may have as many Decimal Places as you pleaſe, in the Root, by proceeding after the ſame Method here ſhew'd.

$$\begin{array}{l} .0\dot{1}4\dot{4}0\dot{0}(.120 = \text{Root.} \\ 2)044 \\ \quad .0000 \end{array}$$

9. In extracting the Square Root of Decimals, you muſt take care to have an even Number of Places, or make them ſo by adding Cyphers toward the Right-Hand, as in the Example. The Cypher in the Root may be omitted, being a Decimal.

10. And in Caſe you want the Square Root of a Vulgar Fraction, if you cannot take the Root of its Terms, as at firſt, then reduce them to lower; and then if you cannot have the Root, reduce your Vulgar to Decimal, and then take the Root as in the 9th Rule above. And thus you have Rules to extract the Square Root of a Whole Number, a Mixt, a Decimal or a Vulgar Fraction. To prove the Extraction, do thus;

11. Multiply the Root in it ſelf, and the Product is the Square Number given, as in the laſt Example .12 by .12 produceth .0144; and ſo of any.

12. But if your Number given be not a compleat Square, but what is called a Surd or

ineffable Number, whoſe Root can never be accurately found, then ſquare what is in the Root, and add to the Product what remain'd of the Square, or rather Surd, given. Thus in the ſecond Example, when there were 3513 in the Root, there remain'd 4509: I do therefore ſquare 3513, and add 4509, and the Sum is the Number given to have its Root extracted, as here appears.

	3513
	3513
	10539
	3513
	17565
	10539
	12341169
Remain'd	4509
Sum =	12345678

13. Or in any Operation you may mark a Surd Number thus $\surd$, or $\overset{2}{\surd}$ before ſuch Number, or expreſs it as a mixt Number.

14. Thus in the 2d Example: I firſt put 3513, and 4 times that more 1 is the Denominator of Fraction, and double the Remainder 4509 is the Numerator; ſo the Root is intirely expreſſed thus, $3513\frac{9018}{13058}$.

The Demonſtration of the Proceſs in Extracting the Square Root.

15. To ſquare 12, which divide into 2 Parts, $10 = a$ and $2 = b$, (*Note*, that $+$ is more, $=$ is equal to) then $a + b = 10 + 2 = 12$, and $a + b$ multiplied in it ſelf produceth $aa + 2ab + bb$, as in the Margin, which is the Canon or General Rule for extracting that Square

Root of any Number. So aa (or a ſquared) is $= 100$, $2ab = 40$, and bb (or b ſquared) is $= 4$, which $= 144 =$ the Square of $a + b$ or $10 + 2$. *Note*, aa and bb is a and b ſquared: So to take the Square Root of 144, proceed as in the Canon and Margin.

For a being $=$ to the Root of aa, I put 10 in the Root, and deduct its Square 100 from 144, and there remains 44 of the Square Number given, *i. e.* $44 = 2ab + bb$.

$$\begin{array}{r}
a+b \\
a+b \\
\hline
aa+2ab+bb \\
100+40+4 \\
=144\,(10=a \\
100=aa \\
\hline
2a=20)\,44\,(2=b \\
\hline
40=2ab \\
\text{more }\ 4=bb \\
\hline
\text{Deduct } 44=2ab+bb \\
\hline
\text{reſts }\ 0
\end{array}$$

Now to ſeparate the $2ab$, ſo as to find the Value of b, I divide 44 by $2a = 20$, and the Quote 2 I put down, it being $= b$; and multiplying $2a$ by b, or 20 by 2, I have $2ab = 40$; to which the Canon directs me to add $bb = 4$, makes $44 =$, as aboveſaid, to $2ab + bb$, which taken from the ſaid Remainder or Dividend 44, the Remainder is 0, and the Root is found as taught at firſt, which agrees with the Proceſs directed by the Canon as is laſt ſhew'd, $10 + 2 = 12$. And thus you may extract the Roots of the Cube, Biquadrate, Surſolid, &c. by only purſuing the Rules of the Canons produced by involving $a + b$ to the 3d, 4th, 5th, &c. Powers.

Sect. 2. *The Extraction of the Cube Root.*

1. A Cube is a ſolid Body bounded by ſix equal Superficies, which being Geometrical Squares, the Side of one of them muſt be the Root of the Cube, or Dimenſion of that Line, which multiplied in it ſelf, gives the Area of one of thoſe equal Squares, and that being multiplied by the ſame Root or Side, (or Depth of the Cube, which is equal to ſuch Root or Side) that laſt Product is the ſolid Content of the Cube.

2. A Table of ſuch Sides, Squares and Cubes (for thoſe Roots which are the 9 Digits) follows.

Roots, or 1ſt Power.	Square, or 2d Power.	Cube, or 3d Power.
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729

3. To find the Side of a Cube whoſe ſolid Content is 1728.

4. You ſee I firſt point over the firſt, and then over every third Figure.

5. Then I conſider what Number being cubed will be next to and leſs than the firſt Branch 1, which is 1; ſo that I put 1 in the Root, and ſubſtracting its Cube from 1, there reſts 0:

6. To that Remainder I bring down my next Branch of the Number given, and it makes 728 for a Reſolvend or Dividend.

$\dot{1}72\dot{8}$(12
33)0728 Reſolvend
3 treb. Root
3 treb. Sq. Root
Sum = 33 the Diviſor
8
12
6
728
0 reſts.

7. To find a Diviſor, I treble the Root 1, makes 3; then I ſquare the Root 1, and treble that Square makes 3, (which place, as you ſee, under Tens Place of the treble Root,) add theſe together, make 33 for your Diviſor to divide 728.

8. Finding that I can have 33 in 72 twice (for as in the Square Root you muſt in this caſe reſerve Units Place of the Dividend) I put 2 in the Root: And,

9. I cube 2 makes 8, as you ſee.

10. I multiply the Square of 2 (laſt put in the Root) by the treble Root 3, makes 12.

11. I multiply 2 in the treble Square of the Root 3, makes 6, which 8, 12 and 6 I place as you ſee in the Example, and the Sum we call a Subtrahend, which taken from the laſt Reſolvend, there remains 0. So the Cube Root is 12 Inches, which is the Side of a cubical Foot, and 1728 is the ſolid Content in Inches.

12. If any more Figures had been in the Number given, you muſt have brought them down to the (0) reſt; and ſo proceed as you ſee farther in this ſecond Example.

987654321.000(= Root 995.8
729 = Cube of 9 in hundreds Place
2457)258654 = 1ſt Reſolvend.
27 = treble the Root 9.
243 = treble Square of 9.
2457 = Sum of the 2 laſt.
729 = Cube of 9 laſt in the Root.
2187 = Square of 9 in treble Root.
2187 = treb. Sq. of the Root in 9.
241299 Sum or Subtrahend.
294327)17355321 = Reſolvend.
297 = treble Root 99.
29403 = treb. Sq. of that Root.
294327 Sum = a Diviſor.
125 = Cube of 5 (in Root.)
7425 = Sq. of 5 in treble Root.
147015 = treb. Sq. of Root in 5
14775875 Sum = Subtrahend.
29703735)2579446000 Reſolvend
2985 treble Root 995.
2970075 treb. Square of Root.
29703735 Sum Diviſor.
512 = Cube of 8
191040 Sq. of 8 in treb. Root.
23760600 treb. Square Root in 8.
2377970912 = Subtrahend.
201475088 reſts.

What is the Cube Root of 987654321: See the Work above.

By the laſt or ſecond Example above, you may ſee,

1. That I put *9* in the Root, becauſe that being cubed, as appears by the little Table of Cubes, is next to and leſs than *9*87, the 1ſt Branch of the Number given, from whence I deduct it, and the Remainder is 258.

2. To that I bring down the next Branch *6*54, and I have 258*6*54 for a Reſolvend or Dividend.

3. For a Diviſor, I add the treble Root *9* = 27 to 3 times the Square of that Root, = 243, and that is a Diviſor.

4. I find I can have 2457 in 258*6*5, *9* times, and therefore I put *9* in the Root.

5. The Cube of the laſt Figure put in the Root (here *9*) more the Square thereof, multiply'd in the laſt treble Root (27) = 2187, more the laſt treble Square of the Root in *9*, *viz.* 2187 added, make 2412*99* for a Subtrahend.

6. I ſubſtract that from the laſt Reſolvend, and the Remainder is 17355, to which bringing down the next Branch 321, makes a new Reſolvend: And ſo proceed as before.

13. For every Decimal Place that you would have in the Root, bring down three Cyphers to the Remainder, as you ſee in the Example.

14. In extracting the Root of a Decimal, you muſt have 3, *6*, *9* or 12, *&c.* Places in the Decimal given, and ſo proceed as in the laſt Example. And,

15. If there be three Cyphers next the Point, you muſt put one Cypher next the Point in the Root.

16. To get the Cube Root of a Vulgar Fraction: If you cannot take the Root in the Terms given, reduce them to lower; and if not then, reduce your Vulgar to a Decimal, and ſo proceed as before directed.

17. The Proof of the 2d Example is as in the Margin, by cubing the Root found.

Example 2. prov'd thus.
995.8
995.8
79664
49790
89622
89622
991617.64
995.8
793294112
495808820
892455876
892455876
987452845.912
Remaind. = 201475088 add
987654321.000 equal
(to the Cube Number
(given for Proof.

18. When you happen to have ſuch Numbers in an Operation whoſe Cube Root cannot be extracted without a Remainder; or if you have not preſent Occaſion to know the Root, you may mark ſuch ſurd Number thus $\sqrt[3]{}$ juſt before the Number, as $\sqrt[3]{17}$, *&c.*

19. *The Demonſtration of the Proceſs in Extraction of the Cube Root.*

Note, That + is more,
= equal to,
× multiply'd by,
aa = *a* ſquared,
bbb = *b* cubed, *&c.*

The Canon directing the Method for extracting the Cube Root, is found by cubing $a+b$; thus, (but this by the by)

$$\begin{array}{rl}
a+b & \\
a+b & \\
\hline
aa+ab\phantom{{}+bb} & \multirow{2}{*}{$\Big\}\text{add.}$} \\
+ab+bb & \\
\hline
aa+2ab+bb & \text{Sum} \\
a+b\phantom{{}+bb} & \\
\hline
aaa+2aab+abb\phantom{{}+bbb} & \multirow{2}{*}{$\Big\}\text{add.}$} \\
+aab+2abb+bbb & \\
\hline
\text{Cube} = aaa+3aab+3abb+bbb & \\
= 9.87654321 \text{ in Example 2.} &
\end{array}$$

$$\begin{array}{rl}
98\dot{7}65\dot{4}32\dot{1} & (900 = a,\ 90 = b \\
729000000 & = aaa \\
2457000)\,258654321 & \text{reſt, divide by } 3a+3ab \\
2700 & = 3a \\
+\,2430000 & = 3aa \\
2457000 & = \text{Sum Diviſor.} \\
729000 & = bbb \\
+\,21870000 & = 3abb \\
+\,218700000 & = 3aab \\
241299000 & = \text{Sum of the 3 laſt} \\
2943270)\,17355321 & = \text{Reſts and now 990 is} = a \\
2970 & = 3a \\
+\,2940300 & = 3aa \\
2943270 & = 3a+3aa \text{ and } 5 = b \\
125 & = bbb \\
+\,74250 & = 3abb \\
+\,14701500 & = 3aab \\
14775875 & \text{Sum of 3 laſt} \\
2579446 & \text{Reſts, and now } 995 = a,
\end{array}$$

and the next Figure put in the Root $= b$.

And thus I have gone over the Canon twice, which you may continue with putting in the Root 8$=b$, &c.

20. Or you may without Decimal Parts expreſs the Root near enough the Truth, by adding 1 to the Sum of the treble Root and treble Square of the Root, which will be the Denominator of the fractional Part, and the Remainder is the Numerator; as the Cube Root of 1740 may be expreſſed $12\frac{12}{469}$.

The Uſe of the Square and Cube Roots will appear in the ſubſequent Parts of the Book.

CHAP. III.

The Menſuration of Superficies and Solids.

THIS being the Foundation of Gauging, it is very proper that it be previouſly learned, and I ſhall ſhew in

Sect. 1. *The Meaſuring of Right-Line Superficies.*

Prop. 1. TO *find the Area of a Geometrical Square.*

Defin. This Figure conſiſts of four equal Sides, which fall perpendicular on each other.

Examp. Suppoſe the Side be 24 Inches, what is the Area.

24
. 24

Anſwer = 576

Rule. Multiply the Side in it ſelf, gives the Area, as here you ſee; where note, that I ſhall throughout uſe the contracted Method of Multiplication, taught under the Rules 6, 7, 8, of Sect. 4. Chap. I.

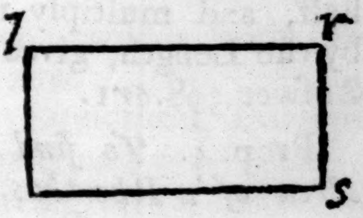
Here it may be obſerv'd, that if the Side were but 12 Inches, the Area would be but 1 Foot; but the Side being 24 Inches, the Area is found = 4 Foot, for 2 times 2 is 4 times as much as 1 time 1.

Prop. 2. *To find the Area of the Parallelogram.*

Defin. This Figure conſiſts of four Sides and four right Angles; but two of the oppoſite Sides are longer than the other 2, but they are parallel, the Longer to the Longer, and the Shorter to the Shorter, like a Table long and ſquare.

Examp. Suppoſe the longer Side $l\,r$ be 24 Foot, and the ſhorter $s\,r$ 2 $\frac{1}{2}$ Foot.

Rule. Multiply the Length by the Breadth, and the Product is the Anſwer.



24
2.5

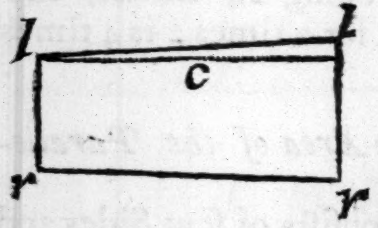
Anſwer = 60.0 Foot

Note, as to Angles, 1. That when a Line, as *s r*, is perpendicular to another Line, as *l r*, the Angle at *r* is right = 90 Degrees, and is the Angle *l r s*: For,

2. All Angles are meaſured by Degrees, or 360 Part of a Circle, and each Degree is 60 Minutes.

3. If an Angle be leſs than a Right, it is called Acute; but if more than a Right, it is Obtuſe; and both theſe are call'd Oblique Angles.

Prop. 3. *To find the Area of a Parallelogramic Figure.*



Defin. This has its two leſſer Sides, as *l r* parallel, and all its four Angles oblique, *i. e.* thoſe at the leſſer End obtuſe, and at the greater acute.

Examp. The greater End is 14 $\frac{1}{4}$, the leſſer 10$\frac{2}{3}$, Length *l c l* = 32.

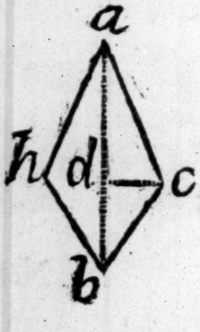
Rule. Add the 2 leſſer Ends together, and take half, and multiply that by the Length, gives the Anſwer 398.6r1.

14.25
10.6r

Sum 24.916r1
Half = 12.4583r1
32

398.6r1

Prop. 4. *To find the Area of a Rhombus.*



Defin. This Figure conſiſts of four equal Sides, and four oblique Angles, the oppoſite Sides parallel, and oppoſite Angles equal.

Examp. The Diagonal *a b* = 32, and the Perpendicular *d c* = 7.5.

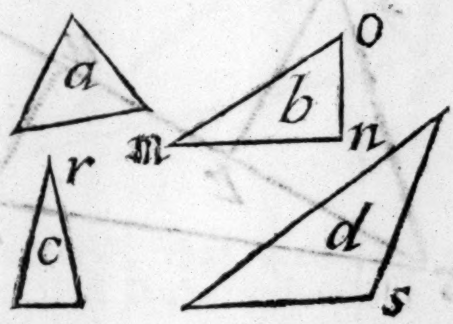
Rule. Multiply thoſe two Lines together, gives the Area 240.

N. B. A Rhomboides has the oppoſite Sides parallel, Angles equal, as the Rhombus; but it has two of the oppoſite Sides longer than the other two. The Way of meaſuring it, is as the Rhombus.

32
7.5
———
240.0

Prop. 5. *To Meaſure Triangles.*

Defin. There are four Kinds of Triangles. Vid. *Euclid* Lib. I. Def. 20. *&c.*



1. The Equilateral, whoſe Sides are equal, as *a*.

2. The Right-angled, which hath one Angle right, as *b*; right at *n*.

3. The Iſoſceles or Equicrural Triangle, having two Sides or Legs equal, as *c*. And,

4. The Scalenum, having all its Sides unequal, and its Angles oblique, as *d*.

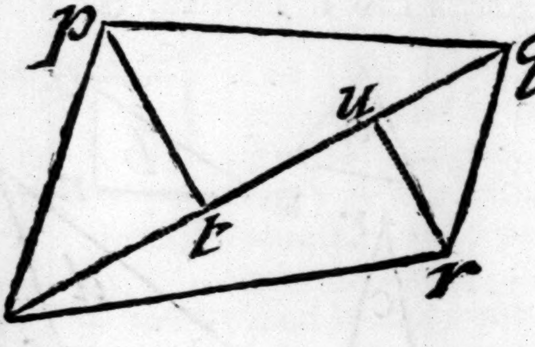
Rule. One Rule meaſureth any of them, which is, Multiply half the Baſe by the Perpendicular; or the contrary, the Baſe by half the other: For all Triangles are half a Square of the ſame Baſe and Perpendicular. And where there are no Perpendiculars, you muſt draw or meaſure one, as in *d*, from *s* to the oppoſite Side; and from *r* in *c*, &c.

Examp. To meaſure the Triangle *b*. The Baſe *m n* is 40, the Perpendicular *n o* = 30; the Area is found by the Rule 600.

30
20
——
600 = the Area.

Prop. 6. *To meaſure the Trapezium.*

Defin. This is an irregular Figure, conſiſting of four Sides and four Angles unlimited.

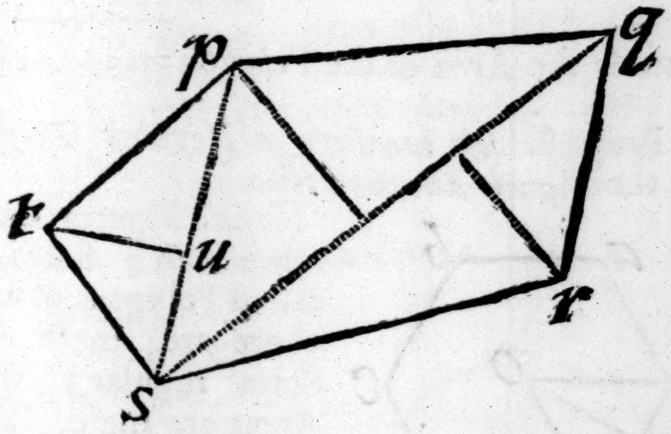


Rule. To meaſure all irregular Figures, which may have never ſo many Angles, we divide them into Triangles: As this is divided into the Triangles *s q p s*, and *s q r s*, and *s q* is the Baſe common to both Triangles; half of which multiply'd by the Sum of the Perpendiculars *p t* and *r u*, or half that Sum by the Baſe, gives the Area.

$s q = 35.75$
$\frac{1}{2}$ is $= 17.875$
$\left.\begin{array}{l} p t = 12 \\ r u = 07 \end{array}\right\}$ Sum $= 19$
——————
Area Anſw. $= 339.625$

Examp. The Baſe *s q* $= 35\frac{3}{4}$, Perpendicular *p t* $= 12$, and *r u* $= 7$; ſo by the Operation the Content is $= 339\frac{5}{8}$; as you ſee.

Prop. 7. *To meaſure any irregular Polygon or many-ſided Figure,* as *p q r s t p.*



You may obſerve, that according to what is ſaid in the laſt Rule, I divide the Figure into three Triangles, or (becauſe two of them have but one common Baſe) I call the Figures, into which the Polygon is divided, a Trapezium *s p q r s*, and a Triangle *s t p s.*

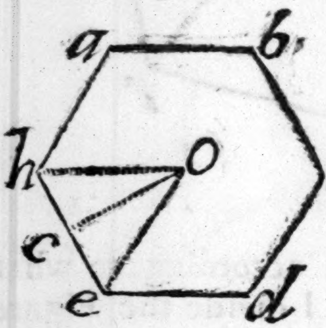
The firſt we will ſuppoſe of the ſame Dimenſions as the 6th Propoſition; to which add the Content of the Triangle, whoſe Baſe *s p* $= 17$, and the Perpendiculer *t u* $= 6$, and the Sum is the Area of the Polygon $390\frac{5}{8}$. See the whole Operation here.

$sp = 17$
Half $= 8.5$
$tu = 6$

So the Area of the Triangle $= 51.0$
Of the Trapezium as under the 6th Prop. $\} = 339.625$

Sum is the Area of the Polygon $= 390.625$

Prop. 8. *To meaſure a Regular Polygon*, as the Figure *a b c d e h*, &c.



Defin. Figures having more than 4 Sides, are called Polygons, of which there are theſe eight Sorts regular; whoſe Areas are theſe, ſuppoſing the Side 1.

The Pentagon having 5 Sides equal, and ſo many equal (Angles.
Area 1.7205 Inch. or .0061 Beer (Gal.

Hexagon	6	2.5981	.0092
Heptagon	7	3.6344	.0128
Octogon	8	4.8284	.0171
Nonogon	9	6.1821	.0219
Decagon	10	7.6940	.0273
Undecagon	11	9.3676	.0332
Dodecagon	12	11.1960	.0397

The Rule to meaſure any of them, is; Find the Area of one Triangle, as under Prop. 5. and multiply that by the Number of Sides, gives the Anſwer.

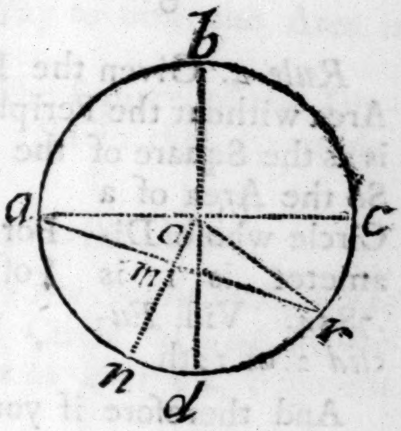
Examp. Of the Hexagon above, ſuppoſe half of one Side, as ce be 30, and the Perpendicular $oc=52$; theſe multiply'd together, give 1560; and there being ſix ſuch Triangles in the Figure, multiply 1560 by 6, gives you the Anſwer 9360.

$oc=52$
$ce=30$
Area $boeb=1560$
6
Area of $abcdeb=9360$

Or you may find the Area by the ſquaring your Side, and multiplying by the Areas reſpectively in the Table.

Prop. 9. *To Meaſure a Circle.*

1. Having the Diameter, to find the Circumference, the Proportion is as 7 to 22, or rather as 1 to 3.1416. So if the Diameter ac be 14, what is the Circumference, Periphery or Ambit $abcda$: See the Operation



7. 22 :: 14.
14

7) 308 (44 Anſwer
28
0

Or thus,
1. 3.1416 :: 14
14

43.9824 = Circumference by Multiplication.

Rule 2. To find the Area, multiply half the Ambit by half the Diameter.

As 22 Or ¼ of the Ambit = 11
by 7 by . . . 14 the Diam.

154 = Area. 154 = Area.

Rule 3. The Circumference given: To find the Diameter. The Proportion is the Converſe of the 1ſt, ſo 44 the Ambit.

As 22. 7 :: 44. 14 = the Diameter. Or.
7 3.1416. 1 :: 44(14.00 =
(the Anſ.

22) 308 (14 = the Anſw. 44.000000
88 125840
17600
0

Rule 4. Given the Diameter: To find the Area without the Periphery. The Proportion is as the Square of the Diameter to its Area: So the Area of a Circle whoſe Diameter is 1 is .7854. Vid. *Euclid* 2. of 12th.

For the Ambit is 3.1416
¼ of the Diameter = .25

Area = .785400

And therefore if you deſire the Area of a Circle whoſe Diameter is 14;

As 1 to .7854: ſo the Square of 14 = 196 to 153.9384.

14 .7854
14 196

196 47124
70686
7854

153.9384 = Area as before *propè*.

Rule. 5. The Area given: To find the Diameter. This is the Converſe of the laſt: As the Area to the Square of the Diameter: Suppoſe the Area 154.

As .7854: (the Area) To 1 (the Square of its [Diameter)
So 154: To 196, whoſe Square Root is 14 [the Anſwer.

.7854) 154.00000 (196 (14 = Root or Diam.
75460 2) 96
47740 0
6160

Rule. 6. Another Way to find the Area is this.

As 14: to 11 :: So the Square of the Diameter: to the Area. So in the Example above, 14 Diameter.

14. 11 :: 196. 154 = the Area.

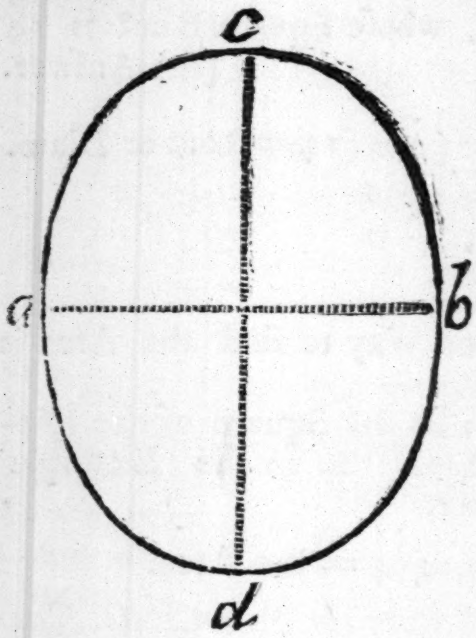
Prop. 10. *To Meaſure the Semi-Circle, Quadrant* (or Quarter of a Circle, as *a o b* in the Circle above) the Sector as *a o r n a*, Segment as *a r n a*.

Rule. One Rule meaſures all, *i. e.* multiply one 4th of the Arch or Curve Line by the Diameter of the Circle, and the Product gives the Area of any Part of a Circle, except the Segment.

2. And, *To find the Area of the Segment of a Circle*, as *a r n a* in the Circle above. Firſt find the Area of the Sector (*a o r n a*) as above, and then find the Area of the Triangle *a o r m a*, which deduct, and the Remain-

der is the Area of the Segment *a m r n a*. But more of this when I come to ſhew the Method of finding the Ullage of a Caſk, and to make the Table of the Areas of Segments, under Prop. 13. Chap. 5.

Prop. 11. *To find the Area of the Ellipſis or Oval.*



Rule. Multiply the *Latus tranſverſum c d*, by the Diameter conjugate *a b*, and the Product is the Square of the Diameter of a Circle equal to the Ellipſis.

Examp. *c d* = 29, *a b* = 22, what the Area.

29
22

638 then ſay,

As 1. .7854 :: 638
638

62832
23562
47124

501.0852 = Area required.

Prop. 12. *To find the Convex Area of a Sphere or Globe.*

Rule. This is four times the Area of a great Circle; therefore,

1. Find the Diameter by the Circumference.

2. Multiply the one by a fourth of the other, gives the Area of a great Circle (if your Ambit was the greateſt.)

3. Multiply the laſt by 4, and you have the Anſwer.

Exam. The Circumference = 22 Diameter 7.

A fourth of 22 = 5.5
multiply by 7

38.5 = Area of a great
by 4 (Circle.

154.0 = the Surface of the
(Sphere.

Prop. 13. *To find the ſuperficial Area of a Cone.*

Rule. As Radius of the Baſe : to the Side of the Cone :: So the Area of the Baſe : to the Area of the Cone.

Or the Area is equal to that of a Triangle whoſe Baſe is the Ambit of the Cone's Baſe, and whoſe Perpendicular is the Side of the Cone. An Example is needleſs.

To find the Area of the Parabola: See *Prop.* 9. *of the next Sect.*

Sect. 2. *Solid Meaſure.*

Prop. 1. *To find the Solid Content of a Parallelepiped.*

Defin. This has a Square or Parallelogram for the Baſe, and whoſe Dimenſions are equal from one End to the other.

Rule. Firſt find the Content of the Baſe or End, and then multiply that by the Length.

Examp. The End is 7.6*r*1 by 4.5, and the Length 15.3. See the Operation.

$$\begin{array}{r}
7.6r \\
4.5 \\
\hline
34.49r1 \\
15.3 \\
\hline
10349r \\
17249r \\
3449r \\
\hline
527.85
\end{array} = \text{Solidity required.}$$

Note, *That a Priſm (which hath a Triangle for its Baſe) is meaſured alſo by multiplying the Baſe by the Length.*

Prop. 2. *To meaſure a Cylinder.*

Defin. This is a ſolid Body having a Circle for its Baſe and a Continuation of equal Circles from one End to the other.

Rule. Find the Area at one End, and multiply that by the Length.

Examp. Diameter 20.5 Length 52 ¼ Inches.

20.5

1025
410

1. .7854 :: 420.25
.7854

168100
210125
336200
294175

Area = 330.06435 at the End.

Area = 330.06435
Length mult. = 52.25

165032175
66012870
66012870
165032175

Anſw. = 17245.8622875

2. But becauſe the Decimal Places may be thought too many, therefore you may perform the Work with much fewer Figures, and have but three Decimal Places in the Product, by the Method under the 5th Rule of Multiplication of Decimals, inverting the Multiplier in working, as there directed, and as in the Example in the Margin, and with 15 Figures leſs.

330.06435
52.25

16503217
660129
66013
16503

17245.862

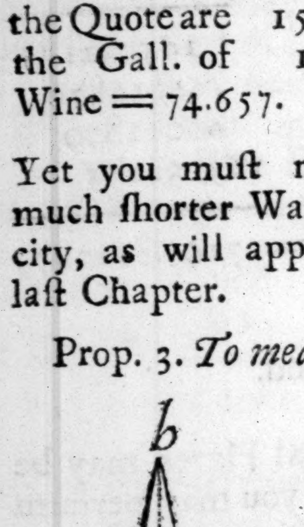
3. But if the Product is to be multiplied or divided by any thing conſiderable with Decimals, then the firſt Product may ſtand: But if you would know how many Gallons of Ale or Beer, a Tub, which is a Cylinder of the ſaid Dimenſions, would contain; you may do it by di-

viding the laſt aboveſaid Content, by 282, the Cubic Inches in a Gallon: Thus,

282)17245.862(61.155 Gallons Beer or
325 (Ale.
And if you 438
divide by 231, 1566
the Quote are 1562
the Gall. of 152
Wine = 74.657.

Yet you muſt not ſuppoſe, but there are much ſhorter Ways to find the Liquid Capacity, as will appear when we come to the laſt Chapter.

Prop. 3. *To meaſure a Cone or Pyramid.*



Defin. A Cone is a Solid which hath a Circle for its Baſe, from whence it is tapering, till it terminate in a Point juſt over the Center of the Baſe, as the Figure *a b d c a*. *Euclid* has demonſtrated, that the Cone is one third of a Cylinder of the ſame Baſe and Altitude, Prop. 10. 12. So that

Rule. Multiply the Area of the Baſe (*a n d m a*) by a third of the Altitude *b c*, and the Product is the Content.

Examp. The Diameter (ſuppoſe the above Circle) 14, the Area is 154, which multiply by a Third of 36.6r (*viz.* of *bc*) =12.2r,

=12.2*r*, and the Product is 1882.2*r*1 = the Content required.

But it may be ſaid, ſeeing the Cone is a ſolid Body, how can we come at the Perpendicular Altitude *b c*?

It muſt be done thus: We can meaſure the Side *a b* = 37.33. And by the Ambit we can find the Semi-Diameter *a c* = 7.

Then by the 47.1 of *Euclid*,

The Square of *a b* 37.33 equal to — 1393.3*r*1
Leſs the Square of *a c* = 7 = . . . 49.

Reſts the Square of *b c* = 1344.3*r*1
Whoſe Sq. Root is = 36.6*r*1 = *b c* required.

Prop. 4. *To meaſure the Fruſtum (or Part) of a Cone*, as (*d s r a d*) in the laſt Figure, whoſe greater Diameter *a d* = 14, the leſſer Diameter (*r s*) = 6.5, Altitude or Depth = 20 Inches.

Rule. Multiply the Areas at the Ends together, and extract the Square Root of the Product; then multiply a Third of the Sum of the two Areas and Root, by the Altitude or Depth; and that Product is the Content of the Fruſtum, as appears by the Work = 1724.45 Inches.

6.5
6.5
1 .7854 :: 42.25 — 33.18315 } 2 Areas
.7854 — 154 }
16900 — 71.485 = Root.
21125
33803 — 258.66815 Sum
29575
33.18315 = Area *r s*.

Area at the leſſer End, as *r s* =33.18315

by Area at the greater End *a d* = 154

13273260

16591575

3318315

5110.205100 Root

14)210 (=71.415

142) 6920

1428) 122451

14296) 814700

99875

86.22271 = $\frac{1}{3}$ Sum.

Multiply by 20 = the Depth.

by 282) 1724.45420 = the Content of the (Fruſtum.

324

And by the 425 (6.11508 Gal. of Beer Anſw.

ſame Rule 1434 See Chap. 5. *Gauging.*

you meaſure 2420

the Fruſtum

of a Pyramid, as Chap. V. Problem 4.

But as was ſaid of the Cone, ſo neither can the true Length of the Fruſtum be taken, if a ſolid Body, but it may be found, *i. e.* (*r t*): Thus,

The homologous Sides of ſimilar Triangles are in Proportion: So,

1. As *a b* : to *c b*, found as before :: So is *a r* : to *t r*. Or,

2. As the Semi-Diameter of the Cone: to its Altitude *c b* :: So is the Semi-Difference of the Diameters of the Fruſtum as (*a t*):

to the true Length of that Fruſtum = tr.

3. Or the Altitude of either the Cone or any Fruſtum may be found by laying down, with a Diagonal Scale, the Diameters and Sides.

Note farther, That the Fruſtum of a Cone may be meaſured alſo thus.

1. Find the ſolid Content of the whole Cone, as Prop. 3.

2. Find the Content of the upper Part, as $rbsr$, by the ſame Prop.

3. Deduct the latter from the former, and the reſt is = the Fruſtum.

Prop. 5. *To meaſure the five regular (or Platonic) Bodys*: Which are

Defin. The Tetrahedron bounded with 4 equal equilateral (Triangles.
Hexahedron - - - - - 6 equal Geometrical Squares.
Octahedron - - - - - 8 equal equilateral (Triangles.
Dodecahedron - - - - 12 equal Pentagons.
Icoſihedron - - - - - 20 equal equilateral (Triangles.

Rule. The firſt of theſe may be meaſured as a Pyramid; finding the Area of one of the Triangles, and multiplying that by a Third of the Altitude.

The ſecond is a Cube; and therefore multiplying one Side in it ſelf, and that Product by the Side again, giveth you the Anſwer.

The third and fifth are 8 and 20 Pyramids, whoſe Points meet in the Center of the Figure: So that having the Area of one of the Triangles, multiply it by one ſixth of the Diſtance between the Middle of two oppoſite Sides, and that Product by the Number of Triangles; as the third by 8, and the fifth by 20.

The fourth is meaſured juſt the ſame way, after you have taken the Area of one of the Pentagons, as taught Prop. 8. of the laſt Sect. foregoing.

Prop. 6. *To find the ſolid Content of a Sphere.*

Defin. A Sphere is a Solid every way round, whoſe Center being imagined to be juſt in the Middle, all Lines extending from thence to the Convex Area, are equal. It is two Thirds of a Cylinder of the ſame Diameter and Altitude, with the Diameter of a Sphere; and thence ariſeth this Rule for meaſuring the Sphere.

Rule 1. Find an Area to the Diameter of a Sphere, which multiply by that Diameter, and two Thirds of the Product is the Content of the Sphere.

Examp. Suppoſe the Diameter 14, then by Prop. 9. &c. of the laſt Sect. the Area will be 154, which multiply by 14, and taking two Thirds of the Product, gives you the Anſwer.

154 = Area of a great Circle.
14 = the Diameter mult.

2156 = the Content of a Cylinder.

718.6r1 = one Third thereof.
2

1437 3r1 = two Thirds, or the Solid Content of the Sphere required.

But by *Euclid* 18. 12. Spheres are in triplicate Proportions of the Diameters.

2. Which Theorem furniſhes us with another way of meaſuring the Sphere, grounded on that foregoing, from which we have two ſtanding Proportionals, *i. e.* as 1 to .5236 : For if the Diameter of a Sphere is 1, the ſolid Content will be .5236. Thus,

1 = the Diameter, 3.1416 = the Ambit.
Which multiply by a 4th of the Diameter } = .25

———

Product .7854 = the Area or So-
of which a Third is = .2618 (lidity of Cylind.
two Thirds = .5236 = the Content of
(the Sphere.

By which the Content under *Rule* 1. laſt mention'd, will be found thus;

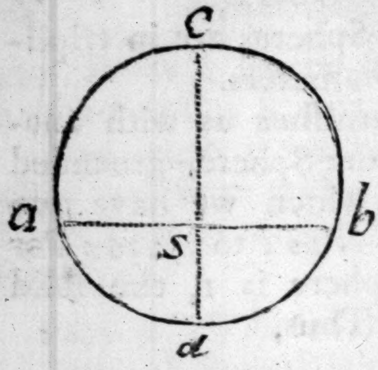
As 1 . .5236 :: 2744 . 1437.3r

That is, as 1, the Cube of the Diameter of the Sphere : is to .5236, that Sphere's Content :: So is the Cube of the Diameter 14 = 2744 : to 1437.3r the ſolid Content of that Sphere.

And ſo is the Cube of any other Diameter of a Sphere to its Content.

14
14
———
196 = Square.
14
———
2744 = Cube of
(14.

Prop 7. *To find the ſolid Content of the Segment of a Sphere.*



Examp. As ſuppoſe of the Segment *a s b d a* of the Sphere *a c b d a*.

But before we ſhew how to find the ſaid Content, there are theſe three Caſes occurrent.

Caſe 1. Having the Altitude of both Segments: To find the Diameter of the Baſe *a b*.

Admit $sc = 8.5$ and $sd = 5.5$ Inches.

Rule 1. As 1 : to 5.5 :: 8.5 : to *a b* ſquared (or $\overline{ab}^2$) $= \sqrt{46.75} = 6.837$.

Caſe 2. Having the Diameter *a b*, and the Altitude of the greater Segment $= sc$: To find the Altitude of the leſſer Segment *s d*.

Rule 2. As $sc = 8.5$: to 1 :: So is $46.75 = \overline{ab}^2$: to $5.5 = sd$.

Caſe 3. Having the Diameter $ab = 6.837$, and the Altitude of the leſſer Segment : To find the Altitude of the greater Segment.

Rule 3. As $sd = 5.5$: to 1 :: So is 46.75 : (the Square of *a b*) to $8.5 = sc$ the Anſwer.

Now to meaſure the leſſer Segment of the Sphere, as ſaid above,

Rule 1. Multiply the Area of the Segment's Baſe by *s d* its Altitude.

2. Multiply that Product by the Sum of half the Altitude of the greater Segment, more a ſixth of the Altitude of the leſſer; and that ſecond Product, divided by the Altitude of the greater Segment, gives us the Content of the leſſer Segment required.

As

Example.

As 1. .7854 :: 46.75
.7854
18700
23375
37400
32725
36.71745 = Area of the Seg. Inch.
5.5 = its Altitude.

201.945975 Product
5.16*r*1 = a 6th of the leſs,
(more half the Alt. of
1211675850 (the greater Segm.

1346306499*r*1 Divide by 8.5 = the
201945975 (Alt. of the *sc.*
1009729875

8.5)1043.38753775(122.751475 = the Con-
193 (tent of the Seg. *asbda*
233 (in Inches.
638 *Note*, If this were a Cop-
437 per Crown of 20 times
125 the Dimenſion, it is
403 but dividing the Con-
637 tent by 282, and the
425 Quote gives you the
0 Gall. of Beer or Ale.

Which is much better than to divide the Area by 282.

Alt. of the greater Segment 8.5
half is = 4.25
Alt. of the leſſer Seg. 5.5, a 6th = 0.916*r*
Sum = 5.166*r* or Mult. Prop.

Prop. 8. *To find the Content of a Spheroid.*

Defin. This Figure is formed by the Rotation of the Semi-Ellipſis, (as *d c a d*, Prop. 11. of the laſt foregoing Section) round its Axis *dc*; and is two Thirds of a Cylinder whoſe Diameter and Length are equal to the Diameter-Conjugate and *Latus tranſverſum* of the Ellipſis.

Exam. Let the Diameter Conjugate be 28, the *Latus tranſverſum* 36; then the Sq. of the Diameter-Conjugate will be 784; Area of that Circle 615.7536; Content of the Cylinder 22167.1296; and two Thirds of that is 14778.0864 = the Anſwer. And if that be divided by 282, the Content of the Spheroid will be 52.4045 Beer or Ale Gallons.

28

28

1. .7854 :: 784

784

31416

62832

54978

615.7536 = Area

36

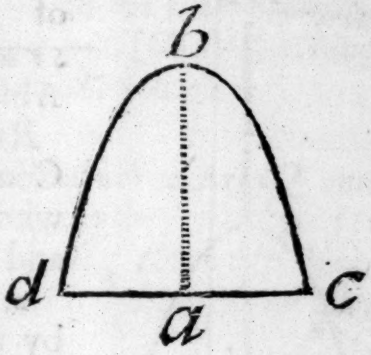
22167.1296 = a Cylin.

7389.0432 = $\frac{1}{3}$

282) 14778.0864 = $\frac{2}{3}$ or the (Content of the Spheroid.

Note, That the Form of Brewers Cask is often the middle Fruſtum of this Figure, which I have ſhew'd four ſeveral Ways of meaſuring in Chap. V.

Prop. 9. *To find the Content of the Parabolic Conoid.*



Defin. This is formed from the Semi-Parabola (*d b a d*) by a Motion round the Abſciſſa (*b a*) and is half of the Cylinder circumſcribed.

Rule. Find therefore the Content of the Cylinder, whoſe Diameter is = *d c*, and its Length = *a b*; and the half thereof is the ſolid Content required.

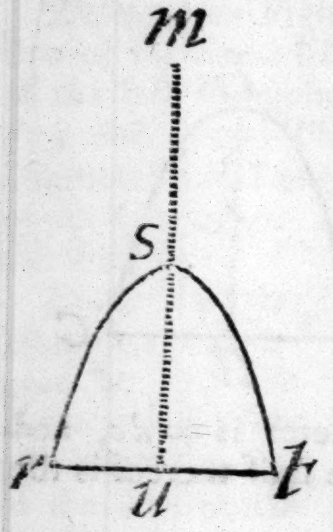
And the Parabola (which is a Superficies, as *d b c d*) is two thirds of a Parallelogram, whoſe Sides, including an Angle, are *a b* and *c d*.

Prop. 10. *To find the Content of the Parabolic Spindle.*

Defin. This Solid is formed by the Rotation of the Semi-Parabola round its Ordinate, and is eight Fifteens of the circumſcribed Cylinder; ſo that finding the Content as a Cylinder, eight Fifteens thereof is the Content.

Prop. 11. *To find the Content of the Hyperbolic Conoid.*

Defin.



Defin. This Solid is formed by the Rotation of the Semi-Hyperbola *s r u s* round its greateſt Abſciſſa (*u s*).

Rule 1. Find the ſolid Content of a Cylinder whoſe Diameter is $= rt$, and its Length $= su$.

2. Multiply that Content by the Sum of three times the *Latus tranſverſum* sm, more two times the Abſciſſa su, and the Product is a Dividend.

3. For the Diviſor, take ſix times the *Latus tranſverſum* sm; more ſix times the Abſciſſa su, and the Quotient ariſing is the Anſwer.

Note, That the Parabola, (whence the Figure under Prop. 9. is formed) is made by cutting the Cone with a Plane, parallel to the Side oppoſite to that wherein the Section is made.

And, That the Hyperbola (whence the Figure under *Prop.* 10. is formed) is made by cutting the Cone more perpendicular; ſo that the Axis or Abſciſſa, if produced upward, would interſect the oppoſite Side of the Cone, at the Diſtance of the *Latus tranſverſum* sm.

Prop. 12. *To find the Content of a Cylindroid.*

Defin. This only differs from the Cylinder, as having an Ellipſis for its Baſe: As ſuppoſe the longeſt Diameter be 36, and the Conjugate 28, and the Length 42.5*r*. See the Work following, where the Anſwer is 33690.5184 Inches, or 119.47 Gallons.

36 =

```
                                36 = Latus tranſ. (or
                                     (longeſt Diam.)
                                28 = Diam. Conjugate (as
                          ———— [Ch.III. §.1. Prop.11.)
    1. .7854 :: 1008. 791.6832
            1008
    ————————————
    791.6832 =Product or Area } multi-
          42.5r1 Length        } ply as
    ————————— (under the 10th Head of
    39584160  (Multiplication of Decim.
    ——————————
     43982399r1
    15833664
    31667328
    ——————————
282) 33690.51839r1 (119.46992 Gallons Beer
                                       (or Ale.
     549
     2670
      1325
       1971
        2798
         2603
          659
```

C H A P. IV.

Farther Inſtructions leading to the Knowledge of the Buſineſs of a Gauger.

CONTAINING, 1. How to find Proprotions of, and Mean Proportions between any Numbers arithmetical and geometrical.

2. Tables of Beer, Ale, Wine and Malt Meaſure.

3. Concerning Gauge-Points; their Calculation and Uſe.

4. The Reduction of one Meaſure to another, ſeveral Ways.

5. Tables ſhewing the Beer, Ale, and Wine Gallons, in any Number of Inches ſolid, by Inſpection, and their Uſe.

Sect. 1. *To find Mean Proportionals between any two Numbers.*

FOR *an arithmetical Mean*, add the two Numbers together, and half the Sum is the Mean: As ſuppoſe between 14 and 20, the Sum is 34, half of which is 17 = the Mean, as 14, 17, 20.

2. Hence it follows, that the double of the Mean is equal to the Sum of the two Extreams. And,

3. That a third Number is found in arithmetical Proportion, by taking the leſſer from double the greater; as in 14 and 17, double of 17 is 34, leſs 14 leaveth 20 for the third Proportional.

Or, add the Difference between the Numbers given to the greater Number, *&c.* will give a third, fourth, *&c.* Proportionals.

4. To find two Mean Proportionals between two Numbers given, add a third of the Difference to the firſt, gives the ſecond, and to that ſecond will give the other Mean; as between 14 and 29, the Difference is 15; a third of that added to 14, gives 19, the firſt, and to 19 makes 24 the ſecond Mean.

Secondly, *For Geometrical Proportionals.*

1. To find a Mean between two Extreams: Multiply the given Numbers together, and extract the Square Root of the Product, as between 9 and 729; the Product of the two Extreams (or firſt and third Numbers) is = 6561; and Square Root of that is 81: So the Numbers are 9, 81, 729.

2. The Product of the two Extreams is equal to the Square of the Mean, as 9 in 729 is = 81 by it ſelf.

3. A third Number is found in proportion to two given, by dividing the Square of the greater by the leſs: As 9 and 81, the Square of 81 is = 6561, which divided by 9, gives 729, the third Number.

4. Three Numbers given, a fourth may be found, which will bear ſuch Proportion to the third, as the ſecond does to the firſt: By multiplying the ſecond and third together, and dividing the Product by the firſt towards the Left Hand; as if 9 is to 81, what is 729 to Anſw. to 6561, proved by the 2d Rule above; for 6561 by 9, is equal to 81 by 729 = 59049. And this is the common *Golden-Rule*.

5. To find two Means between two Extreams, as between 9 and 6561: Divide the greater by the leſs, gives 729; the Cube Root of which is 9, by which multiplying the firſt Number, gives 81, and that by 9, gives the other Mean. And ſo 20 and 80 are the Means between 5 and 320.

Sect. 2. *Tables of Beer, Ale, Wine and Malt-Meaſure.*

NOTE, that the Beer and Ale Gallons are the ſame, each 282 Inches; but the Beer Barrel is 36 Gallons, and the Ale but 32, within the Weekly-Bills; and 34 Gallons each without the Bills.

A Table of Wine-Meaſure.

Tuns.	Pipes.	Hogſheads.	Gallons.	Quarts.	Pints.	Inches ſolid.
					1	28⅞
				1	2	57¾
			1	4	8	231
		1	63	252	504	14553
	1	2	126	504	1008	29106
1	2	4	252	1008	2016	58212

Note, There are ſome other Denominations of this Meaſure of leſs Uſe; as the Aum = 42 Gallons, or the Tertian of a Pipe; the Puncheon = 2 Aums, or 84 Gallons; and one Tierce (or Tertian) and a half is 63 Gallons = the Hogſhead; and the Rundlet is about 18 Gallons.

A Table of Beer-Meaſure.

Note, *This is the Beer Barrel within the Bills of Mortality. And that in all other Places 'tis but* 34 *Gallons or* 9588 *Inches.*

Barrels.	Kilderkins.	Firkins.	Gallons.	Pottles.	Quarts.	Pints.	Inches. ſolid.
						1	35¼
					1	2	70½
				1	2	4	141
			1	2	4	8	282
		1	9	18	36	72	2538
	1	2	18	36	72	144	5076
1	2	4	36	72	144	288	10152

A Table of Ale-Meaſure.

Note, *This is the Ale Barrel within the Weekly-Bills: And that in all other Places the Barrel is* 34 *Gal.* = 9588 *Inches as the Beer Barrel is in thoſe Places.*

Barrels.	Kilderkins.	Firkins.	Gallons.	Pottles.	Quarts.	Pints.	Inches ſolid.
						1	35¼
					1	2	70½
				1	2	4	141
			1	2	4	8	282
		1	8	16	32	64	2256
	1	2	16	32	64	128	4512
1	2	4	32	64	128	256	9024

Theſe Tables are eaſy: For Example in Beer-Meaſure, 1 under Gallons (caſting your Eye towards the Right-Hand) ſhews there are therein 2 Pottles, 4 Quarts, 8 Pints, and 282 Inches: And in 1 Barrel there are 2 Kilderkins, 4 Firkins, 36 Gallons, 72 Pottles, 144 Quarts, 288 Pints, 10152 ſolid Inches, &c.

A Table of Malt-Meaſure.

Quarters.	Buſhels.	Pecks.	Gallons.	Pottles.	Quarts.	Pints.	Inches ſolid.
						1	$33\frac{60059375}{1000000000}$
					1	2	$67\frac{2007875}{10000000}$
				1	2	4	$134\frac{401575}{1000000}$
			1	2	4	8	$268\frac{80315}{100000}$
		1	2	4	8	16	$537\frac{6063}{10000}$
	1	4	8	16	32	64	$2150\frac{4252}{10000}$
1	8	32	64	128	256	512	$17203\frac{4016}{10000}$

By this Table it appears, that in 1 Quarter of Corn or Malt, are 8 Buſhels, 32 Pecks, 64 Gallons, 128 Pottles, 256 Quarts, 512 Pints, and 17203.4016 ſolid Inches; and in the like Manner read the reſt, which are here placed with the greateſt Eaſe to be underſtood, far exceeding other Tables of the kind, in that Reſpect.

Sect. 3. *Concerning Gauge-Points.*

1. GAuge-Points are the Diameters of Circles which contain ſome known Meaſure (commonly thoſe moſt uſed) upon one Inch in Depth of a Veſſel: As the Gauge-Point for Wine-Meaſure is the Diameter of a Circle, whoſe Area upon one Inch deep is a Gallon; and the like may be ſaid of the Beer or Ale-Gallon, or Barrels, as ſhall be demonſtrated.

2. Theſe Gauge-Points are uſed in finding the Areas of Circles, in Liquid-Meaſure, to one Inch deep; whether thoſe Circles be more or leſs in Diameter than the Gauge-Points, which are of great Uſe in inching of Veſſels, *i. e.* in making Tables which ſhew the Content of any Veſſel, at every Inch in depth from the Top to the Bottom.

3. Having thus ſhewn what Gauge-Points are, and their Uſe in general, I come next to ſhew how to find them, with ſomewhat of their Uſe, (and in the next Chapter ſhall ſhew their more particular Uſe.) As,

To find the *Gauge-Point for Wine*, or the Diameter of a Circle whoſe Area is a Wine-Gallon. *Vid.* Eucild 2. 12.

As .7854, the Area of a Circle is: to 1, the Square of its Diameter :: So is 231, the ſolid Inches in a Wine-Gallon: to the Square of the Diameter of that Circle. Thus,

As .7854 : 1 : : 231.00000000 : 294.1176
) 73920 (294.1176 = the Square of
32340 the Gauge-Point whoſe
9240 Root is 17.15, the
13860 Gauge-Point for a
60060 Wine-Gallon.
50820

4. To find the Gauge-Point for a Beer or Ale-Gallon, which is 282 ſolid Inches: Therefore,

As .7854 : 1 :: 282 : to 359.0527
) 282.00000000 (= 359.0527 the
46380 Square of the Gauge-
71100 Point whoſe Root is
41400 = 18.95 *ferè*, the
21300 Gauge-Point for
55920 the Beer or Ale
Gallon.

5. To find the Gauge-Point for the Malt Buſhel, which by the 12th of Queen *Anne*, is 2150.4252 ſolid Inches: Therefore,

As .7854 : 1 :: 2150.4252 : to 2738
) 57962 (2738 = the Sq. of the
29845 Gauge-Point, whoſe
62832 Root is 52.3259.

6. To find the Gauge-Point for the Malt Gallon, which is the 8th Part of the Buſhel, or 268.80315 ſolid Inches: So that,

As .7854 : .1 :: 268.80315 : to 342.25.

) 35183 (342.25 = the Square
17671 of the Gauge-Point,
19635 whoſe Root is 18.5
39270 Inch. = the Gauge-Point.

7. For Maſh-Tubs the Gallon was formerly reckoned 272.25 Inches: And the old experienced Exciſe-Officer, Mr. *Michael Dary*, and others after him, eſtimated, that Malt thorough wet, would be compreſſed or contracted into a leſſer Bulk, by a 6th Part: So that they make the Maſh-Gallon but 227 Inches (which is 272 leſs 45 = 227.) So that the Gallon, according to the aboveſaid Statute under the 5th and 6th Rules, being 268 Inches, one 6th being deducted (or 44) there reſts, for the preſent Maſh-Gallon but 224 Inches; the Gauge-Point for which is as follows.

As .7854 : 1 :: 224 : to 285.205

) 224.00000000 (285.205 *ferè* =
66920 the Square of the
40880 Gauge-Point,
16100 whoſe Root is
39200 16.888 the Gauge-Point.

But I am informed by one, who knows the Practice very well, that they only now gauge the Liquor without the Maſh or Grains; and it was always my Opinion (who have been converſant in the Art of Gauging, near 40 Years) that it was the beſt Way.

8. To find the Gauge-Point for a Barrel of Beer.

By the Table of Beer-Meaſure above, there are in a Barrel 10152 ſolid Inches: And therefore,

As .7854 : is to 1 :: So is 10152 : to 12925.8975.

.7854) 10152 (12925.8975 = the Square of
2298.0 the Gauge-Point, whoſe
7272.0 Root is 113.69, the
2034.0 Gauge-Point for the
4632.0 Beer-Barrel.
70500
76680
59940
43820

9. To find the Gauge-Point for a Barrel of Ale, which is 9024 Inches.

As .7854 : 1 :: 9024.0 : 11489.6867
) 11700(11489.6867=the Square
38460 of the Gauge-Point,
70440 whoſe SquareRoot
76080 is 107.19 = the
53940 Gauge-Point required.
68160
53280
61860

	Square of the Gauge-Pts.	Gauge-Pts.
So that for the Wine-Gallon you have	294.1176	—17.15 Inches.
For the Beer or Ale-Gallon	359.0527	—18.95 *ferè.*
For the Malt-Gallon, as now,	342.25	—18.5.
Ditto as formerly reckoned,	346.6386	—18.62 *ferè.*

For

	Square of the Gauge-Pts.	Gauge-Pts.
For the Malt-Buſhel, as *per* Act above,	2738.	—52.3259.
Ditto as formerly accounted,	2773.109	—52.66.
Maſh-Tub Gallon, as *per* 7th Example,	285.205	—16.888.
Ditto as formerly eſteemed,	289.0247	--17.
Beer-Barrel *within the Weekly-Bills*,	12925.8975	-113.69.
Ale-Barrel *within the Weekly-Bills*,	11489.6867	-107.19 Inches.
Beer-Barrel *without the Weekly-Bills*,	12207.7922	-110.48.
Ale-Barrel *without the Bills of Mortality*,	*ditto*	

10. And by theſe Squares of Gauge-Points, you may find the Content of one Inch deep of any round or elliptical Veſſel, whether the Diameter be more or leſs than the Gauge-Points. *For Example*, To know how much Beer a Veſſel that is 35 Inches Diameter will contain upon one Inch deep: This is one of the moſt common Caſes in Gauging: And,

The General-Rule is,

As the Square of the Gauge-Point : is to one Gallon :: So is the Square of any other Diameter, as here of 35 : to the Area of one Inch deep.

i. e. As 359.05 : 1 :: 1225 : 3.4118 Gallons one
(Inch deep.

Or if the Diameter be but 12 :: So is 144 : to .4010 Gallon, and ſo others.

11. If the Tun be elliptical, multiply the Croſs-Diameters together, and divide by the Square of the Gauge-Point, and the Quote is the Area of one Inch deep. *Example*, Suppoſe the longeſt Diameter be 29, and the Conjugate 22.

As 359.05 : 22 :: 29 : 1.776
29

)638.00000(1.776 Gallon Beer or Ale of one Inch deep, of that Veſſel or Tub.
278950
276150
248150

12. If the Veſſel is a Square, or bounded with right, not curved Lines, as a Parallelepipedon, Chap. III. Sect 2. Prop. 1.

If you are to find the whole Content, multiply the Area at one End, (found as by the 1ſt or 2d Propoſitions of Chap. III. and Sect. 1.) and multiply that by the depth; then divide the Product by 282, the Quote gives the Anſwer in Beer or Ale-Gallons.

But if it be required to inch this Veſſel, or to find the Content when Part full, then divide the Area at one End, by 282, and whatever you find the depth to be, multiply that Quote or Area in Gallons, by the Depth, gives the Content. Of this I ſhall give Examples in the fifth Chapter next following.

Sect.

Sect. 4. *The Reduction of Meaſures from one to another.*

1. TO reduce Beer-Gallons into Barrels, divide by 36, or multiply by .027*r1*. So in 432 Gallons is 12 Barrels: Example of the ſecond way, becauſe of the repeating Digit, is in the Margin.

432
027*r*

add 3024

is = 3359*r1* } add
864

Sum = 11.999*r1* = 12 Bar.

2. To reduce Gallons of Ale into Barrels, divide by 32, or multiply by .03125; by either of which Ways, for Example, you will find, that in 352 Gallons there are 11 Barrels.

3. To reduce any Number of ſolid Inches into Gallons of Wine, Beer or Ale.

For Wine divide by 231, or multiply by } .00433
For Beer or Ale divide by 282, or multiply by } .003546

So in 22869 Inches is 99 Gallons of Wine,
Or 81.1 *fere* of Beer or
(Ale, by either Way.

Or, you have Tables in the next Section, to do theſe or the like by Inſpection.

4. To reduce Barrels of Ale to Barrels of Beer, Multiply by 8, and divide by 9, or multiply by 8*r*.

So in 99 Barrels of Ale are 88 Barrels of Beer, by either Way.

5. To reduce Barrels of Beer into Barrels of Ale.

Multiply by 9, and divide by 8, or multiply by 1.125.

So in 88 Barrels of Beer are 99 Barrels of Ale by either Method.

Sect. 5. *Table ſhewing the Beer, Ale and Wine-Gallons in any Number of ſolid Inches, by Inſpection; with the Uſe of thoſe Tables.*

The Uſe of the following Tables.

1. TO find how many Gallons is in any Number of ſolid Inches: As ſuppoſe in 14672, how many Gallons of Beer or Ale? Look in the Table for the next leſs, and deducting it, you have 8 Inches remains; and againſt the next leſs in the Table ſtands 52 Gallons. So the Anſwer is 52 Gallons, 0 Pints, and near a 4th of a Pint.

14672 = the N° given
14664 = next leſs in the Table.
———
8 reſts Inches.

If a greater Remainder had happened, you might have compared it with the Denominations under a Gallon in the Table of Beer-Meaſure foregoing. As,

2. *Example*, In 28120 Inches, are 99 Gallons and 202 Inches, which by the foregoing Table is 1 Pottle, 1 Pint, and above $\frac{2}{3}$ of a Pint.

A

A Table of the Gallons of Beer or Ale in any Number of ſolid Inches.

Solid Inches.	Gallons Beer or Ale.	Solid Inches.	Gallons Beer or Ale.	Solid Inches.	Gallons. Beer or Ale.
564	2	9870	35	19176	68
846	3	10152	36	19458	69
1128	4	10434	37	19740	70
1410	5	10716	38	20022	71
1692	6	10998	39	20304	72
1974	7	11280	40	20586	73
2256	8	11562	41	20868	74
2538	9	11844	42	21130	75
2820	10	12126	43	21432	76
3102	11	12408	44	21714	77
3384	12	12690	45	21996	78
3666	13	12972	46	22274	79
3948	14	13254	47	22560	80
4230	15	13536	48	22842	81
4512	16	13818	49	23124	82
4794	17	14100	50	23406	83
5076	18	14382	51	23688	84
5358	19	14664	52	23970	85
5640	20	14946	53	24252	86
5922	21	15228	54	24534	87
6204	22	15510	55	24816	88
6486	23	15792	56	25098	89
6768	24	16074	57	25380	90
7050	25	16356	58	25662	91
7332	26	16638	59	25944	92
7614	27	16920	60	26226	93
7896	28	17202	61	26508	94
8178	29	17484	62	26790	95
8460	30	17766	63	27072	96
8742	31	18048	64	27354	97
9024	32	18330	65	27636	98
9306	33	18612	66	27918	99
9588	34	18894	67	28200	100

II. A Table of Gallons of Wine in any Number of ſolid Inches.

Solid Inches.	Gallons Wine.	Solid Inches.	Gallons Wine.	Solid Inches.	Gallons Wine.
462	2	8085	35	15708	68
693	3	8316	36	15939	69
924	4	8547	37	16170	70
1155	5	8778	38	16401	71
1386	6	9009	39	16632	72
1617	7	9240	40	16863	73
1848	8	9471	41	17094	74
2079	9	9702	42	17325	75
2310	10	9933	43	17556	76
2541	11	10164	44	17787	77
2772	12	10395	45	18018	78
3003	13	10626	46	18249	79
3234	14	10857	47	18480	80
3465	15	11088	48	18711	81
3696	16	11319	49	18942	82
3927	17	11550	50	19173	83
4158	18	11781	51	19404	84
4389	19	12012	52	19635	85
4620	20	12243	53	19866	86
4851	21	12474	54	20097	87
5082	22	12705	55	20328	88
5313	23	12936	56	20559	89
5544	24	13167	57	20790	90
5775	25	13398	58	21021	91
6006	26	13629	59	21252	92
6237	27	13860	60	21483	93
6468	28	14091	61	21714	94
6699	29	14322	62	21945	95
6930	30	14553	63	22176	96
7161	31	14784	64	22407	97
7392	32	15015	65	22638	98
7623	33	15246	66	22866	99
7854	34	15477	67	23100	100

CHAP. V.

Contains the beſt Methods of Gauging any Veſſel or Cask, &c.

1. HOW to gauge and inch any ſquare, round or elliptical Veſſel, which is not tapering, but of the ſame Dimenſions throughout.

2. To gauge and inch ſquare, circular and elliptical Veſſels, which are tapering, or Fruſtums of Cones or Pyramids.

3. How to gauge and inch a Copper, and allow for the Crown.

4. To gauge Casks which lie with their Axis parallel to the Horizon, four ſeveral Ways; and to find the Content when part full, or the Ullages, with Tables of the Segments of Circles, and a new Way to find the Content, without ſuch Tables.

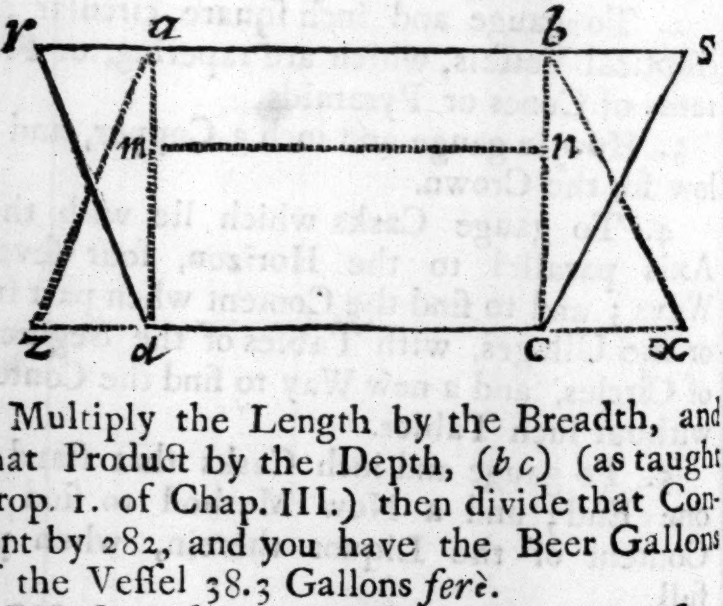
5. To gauge and inch Casks that ſtand on one End; and a New Method to find the Content of the Liquor therein, when part full.

6. A Table of Cylinders, and its Uſe in Gauging round or elliptical Veſſels.

7. Of the Gauging of Malt in the Ciſtern, Couch and Floor.

Having, in the third Chapter, laid ſo ſubſtantial a Foundation of Gauging, by Definitions of every Kind of Superficies and Solids, with Rules and Examples of their Menſuration, I have, I hope, ſufficiently prepared the diligent Reader for the underſtanding of what is here to follow; which is the whole Body of the Art of Gauging.

Prob. 1. *To find the Content of a Veſſel in Gallons, which has its Baſe near ſquare, but ſomewhat longer than broad, and are the ſame from one End to the other*, viz. 18 *by* 25, *and the Depth* = 24 *Inches*, as the Figure *a b c d*, or Parallelepiped.



Multiply the Length by the Breadth, and that Product by the Depth, (*b c*) (as taught Prop. 1. of Chap. III.) then divide that Content by 282, and you have the Beer Gallons of the Veſſel 38.3 Gallons *ferè*.

But ſuppoſe this Veſſel is but very often part full, and I come and find it but *m d* or *n c* of Liquor = 20 Inches.

In this Caſe find the Area in Gallons, and keep a Note thereof, and multiply it by what you find the Depth to be at any time: So here it will be found to contain 31.914 Gallons of Beer, the Area being 1.5957, multiplied by 20, the Depth. And you may make a Table with eaſe, of its Content, at every Inch deep; and then when you come, you have nothing to do but take the Depth of the Liquor, and look in your Table for the Content.

Prob. 2. *To gauge a round Veſſel or Cylinder, ſuppoſe* a b c d, *whoſe Diameter* a b *or* c d *is* 25, *Depth* a d *or* b c = 24.

Firſt find the Area at one End, as taught Rule 10. of Sect. 3. of Chap. IV. For,

As 359.05 : to 1 Gallon :: So is 625 : (the Square of 25) to 1.74 Gallons the Area, which multiply'd by the Depth 24, gives 41.77 Gallons the Content.

And this Veſſel is inched, by only multiplying the Area 1.74, by the ſeveral Inches that the Veſſel is in Depth, and ſetting the ſeveral Products againſt the ſeveral Depths by which you multiplied; and then you have nothing to do, but when you come, take the Depth of the Liquor, and look in your Table, where you'll find the Content; which is ſo eaſy, that it would reflect on the Reader's Ability, (after all that has been ſaid before:) To give Example,

Prob. 3. *Admit the Baſe* d c, *were an Ellipſis, whoſe greateſt Diameter* a c *were* = 25, *and the Conjugate Diameter* 19, *Length* 24, *thro' which Dimenſion the ſame Diameters are continued.* For Anſwer ſay,

As 359.05 : to 1 :: So the Prroduct of the two Diameters 475 : to 1.323, the Area upon one Inch deep, which multiply'd by 24, gives 31.752 Gallons of Beer or Ale, the Content. And, as was ſaid of the laſt, if you multiply the Area 1.323 by the ſeveral Inches, that your Veſſel is deep, it will give you a Table to have recourſe to at any Time, for any Depth of Liquor that you may find; which is too plain to need an Example of Inching.

Prob. 4. *To gauge and inch a tapering Veſſel; as ſuppoſe in the above Figure* r s c d r, *the Fruſtum of a Pyramid, whoſe lower Baſe is the ſame as under Prob.* 1. = 18 *broad, and* 25 *long; and admit the greater End be* 33 *long and* 26 *broad, and the Depth* a d *or* b c = 24, *what is the Content of the whole, and of every Inch of the Depth of the Veſſel?*

(1.) Firſt for the Content of the whole, according to the Rule given for the Fruſtum of a Cone, Chap. III. Sect. 2. Prop. 4.

33 | 25
26 | 18

858 Area of the (greater Baſe. | 450 = Area of the (leſſer Baſe

So the Content of the Whole is 54.72 Gallons.

858
450 } multiply

Extract the Root of 386100 (621 = Root.
12) 261 | 858
124) 1700 | 450 } Areas.
459
1929 Sum.

282) 15432.00 (54.72 Gal. ⅓ is = 643
1332 (Beer. | by 24 deep.
2040
660 | Content 15432 Inches.

(2.) I ſhall now ſhew how many Gallons this Veſſel contains at any Number of Inches, in Depth of Liquor; but ſhall firſt give this preparatory Scheme.

The preparatory Scheme or Table for Inching.

Depth of the Veſſel the Middle of every 6 Inches.	Mean Dimenſions at thoſe Depths.		Products or Mean Areas	Mean Areas in	Content of every 6 Inches of Depth in
Inches.	Len.	Bred.	Inches.	Gallons.	Gallons.
21	32 by	25	800	2.8368	17.0208
15	30	23	690	2.4468	14.6808
9	28	21	588	2.0851	12.5106
3	26	19	494	1.7517	10.5102
Sum or Content of the Veſſel					54.7224

I ſhall next ſhew the making of theſe Columns: And,

1. That next the Left-Hand is the Depth of the Veſſel of the firſt 6 Inches next the top of it, or an arithmetical Mean between 18 and 24; as 15 is the Mean between 12 and 18, 9 between 6 and 12, *&c.*

2. The ſecond Column is the Length and Breadth at thoſe Depths, which are beſt found by the Pen, not by an Inſtrument, as a great Man adviſes. I find them thus:

As 24: (the whole Depth of the Veſſel) is to 8 (the Difference between the Length at the Top and Bottom) :: So is 21: to 7; which added to 25 (the Length at the bottom) makes 32, the Length againſt 21; or added to 18, the Breadth at the Bottom, makes

25 the Breadth againſt 21; and ſo of the reſt of the Lengths and Breadths. For,

As 24 : 8 :: 15 : 5, more 25=30 the Length at (15 deep.
and ſo 9 : 3, more 25=28 againſt 9.
and ſo 3 : 1, more 25=26 which is a-(gainſt 3, *&c.*

3. The third Column is the Products of the Length multiplied by the Breadth, which ſtands againſt it, as 32 by 25 is 800, *&c.*

4. The fourth Column is the third divided by 282, and ſo in Gallons of Beer.

5. The fifth Column is the fourth multiplied by 6; that is to ſay, every ſeveral Mean by 6, which Products muſt be the Content of every 6 Inches deep, and conſequently the Sum muſt be the Content of the whole Veſſel; as you ſee it is to a Nicety by what is ſhewed above.

(3.) I ſhall now, by help of the ſecond Column from the Right-Hand of the Table laſt foregoing, ſhew how to inch the Veſſel above, *viz. r s c d, as it ſtands on the leſſer Baſe c d*, of the Figure laſt above.

Inches deep of the Veſſel's Liquor.	Content at thoſe Inches deep Gal. as Beer or Ale.	Proof of the Work. as you proceed.
24	54.7224	
	2.8368	deduct continually for 6 (Inches.
23	51.8856	
22	49.0488	17.0208
21	46.2120	37.7016=Proof.

Inches deep of the Veſſel's Liquor.	Content at thoſe Inches deep Gal. as Beer or Ale.	Proof of the Work as you proceed.
20	43.3752	
19	40.5384	
18	37.7016	
	2.4468	deduct for the next 6.
17	35.2548	
16	32.8080	
15	30.3612	14.6808
14	27.9144	23.0208 Proof.
13	25.4676	
12	23.0208	
	2.0851	deduct for the next 6.
11	20.9357	
10	18.8506	12.5106
9	16.7655	10.5102 Proof.
8	14.6804	
7	12.5953	
6	10.5102	
	1.7517	deduct for the reſt.
5	8.7585	
4	7.0068	
3	5.2551	
2	3.5034	
1	1.7517	Proof being equal to the
0	0.	(laſt deducted Number,
		(or Subducend.

Note, The Way I prove every 6 Numbers that they are truely deducted, is, as in the firſt Example. I multiply the deducted Number by 6, gives 17.0208, which deduct from the 54.7224, and the reſt is equal to the laſt Number 37.7016, which ſtands againſt 18. And ſo of any.

You will eaſily ſee that I deduct the firſt Mean Area from 54.7224, and from the Remainder, till I come to 18 Inches, and then I deduct 2.4468, the ſecond Mean Area, till I come to 12 Inches, and ſo proceed with the reſt of the Mean Areas in the preparatory Table: And after I have deducted the fourth Mean Area, that Number reſting proves all right.

(4.) *An Example of Inching this Veſſel, if it ſtood on the greater Baſe* z x, *of Fig.* a b x z a.

Inches deep of Liquor.	Content Gal. Beer.	Proof of the Work as you proceed.
24	54.7224	
	1.7517	deduct for 6 Inches of (Depth.
23	52.9707	
22	51.2190	10.5102
21	49.4673	44.2122 Proof.
20	47.7156	
19	45.9639	
18	44.2122	
	2.0851	deduct for the next 6.
17	42.1271	
16	40.0420	12.5106
15	37.9569	31.7016 Proof.
14	35.8718	
13	33.7867	
12	31.7016	
	2.4468	deduct for the next 6.
11	29.2548	
10	26.8080	14.6808
9	24.3612	17.0208 Proof.
8	21.9144	
7	19.4676	
6	17.0208	
	2.8368	deduct for the reſt.

5 14.

Inches deep of Liquor.	Content Gal. Beer.	Proof of the Work as you proceed.
5	14.1840	
4	11.3472	
3	8.5104	
2	5.6736	
1	2.8368	Proof the whole, being
0	0.	(the laſt deducted N°.

Prob. 5. *To gauge and inch a circular tapering Veſſel, which is the Fruſtum of a Cone.*

The Menſuration of a Solid in this Form, is ſhewed Prop. 4. of Sect. 2. of Chap. III. And ſuppoſe that a Veſſel ſtanding on its greater Baſe, and that the Dimenſions are,

Greater Diameter 70 Inches } Depth of the
Leſſer Diameter 56 } Veſſel 36 Inch.

What is the Content of the whole, and at every Inch of the Depth?

By the preparatory Table following, the whole is 399.4857 Gallons, agreeing to leſs than a 10th of a Gallon, with the Method in Prop. 4. laſt above quoted or referred to: This ſtanding on its greater Baſe.

The Preparatory Scheme or Table for Inching this Veſſel.

Inches from the Top.	The Middle of every 9 Inches in Depth.	Mean Diameters at thoſe Depths in Inches.	Mean Areas at thoſe Diameters in Gal. Beer or Ale.	Content of every 9 Inches of Depth in Gallons.
1ſt. 9	31.5	57.75	9.2885	83.5965
2d. 9	22.5	61.25	10.4485	94.0365
3d. 9	13.5	64.75	11.6768	105.0912
4th. 9	4.5	68.25	12.9735	116.7615
Sum or Content of the Veſſel =				399.4857

1. To make the firſt Column next the Left-Hand, I deduct 4.5 Inches from 36, and there remains 31.5 = the Middle of the firſt 9 towards the Top; then I deduct 4.5 from 27, gives 22.5, the Middle of the ſecond 9 Inches; and from 18 and 9 leaves 13.5, and 4.5 the other Means.

2. For the Diameters at thoſe Depths: I ſay,

As 36, the whole Depth : is to 14 the Difference of the given Diameter :: So 31.5 : to 12.25, which deducted from the greater Diameter = 70, there reſts 57.75, the Diameter at the Middle of the firſt 9 Inches. And ſo is 22.5 : to 8.75, which deduct from 70, there remains 61.25 = the Diameter at the Middle of the ſecond 9 Inches, *&c.*

3. For the Areas in the third Column, from the Left-Hand :

As 359.05 : (the Square of the Gauge-Point) is to 1 Gallon :: So is 57.75 ſquared : to 9.2885 the Area at 31.5 Inches deep; and ſo are the reſt of the Areas found.

4. Theſe mean Areas, multiply'd by 9 give the reſpective Contents at 9 Inches of Depth. And the Sum of theſe is the whole Content of the Veſſel as aforeſaid: And what the Content at every Inch deep is, follows.

Inches deep of Liquor	Content at thoſe Inches deep in Gal. Beer or Ale.	Proof of the Work. as you proceed.
36	399.4857	
	9.2885	deduct for the firſt 9 (Inches deep.
35	390.1972	
34	380.9087	

Inches deep of Liquor.	Content at thoſe Inches deep in Gal. Beer or Ale.	Proof of the Work as you proceed.
33	371.6202	
32	362.3317	83.5965
31	353.0432	315,8892 Proof = 27
30	343.7547	(deep.
29	334.4662	
28	325.1777	
27	315.8892	
	10.4485	deduct for the ſecond 9
26	305.4407	(Inches.
25	294.9922	
24	284.5437	
23	274.0952	94.0365
22	263.6467	221.8527 Proof = 18
21	253.1982	(deep.
20	242.7497	
19	232.3012	
18	221.8527	
	11.6768	deduct for the third 9 Inch.
17	210.1759	
16	198.4991	
15	186.8223	
14	175.1455	105.0912
13	163.4687	116.7615 Proof = 9
12	151.7919	(deep.
11	140.1151	
10	128.4383	
9	116.7615	
	12.9735	deduct for the fourth 9
8	103.7880	(Inches.
7	90.8145	

Inches deep of Liquor.	Content at thoſe Inches deep in Gal. Beer or Ale.	Proof of the Work as you proceed.
6	77.8410	116.7615 =
5	64.8675	116.7615 = 9 times
4	51.8940	(12.9735
3	38.9205	
2	25.9470	
1	12.9735	Proof of the whole = the
0	0.	(laſt deducted Number.

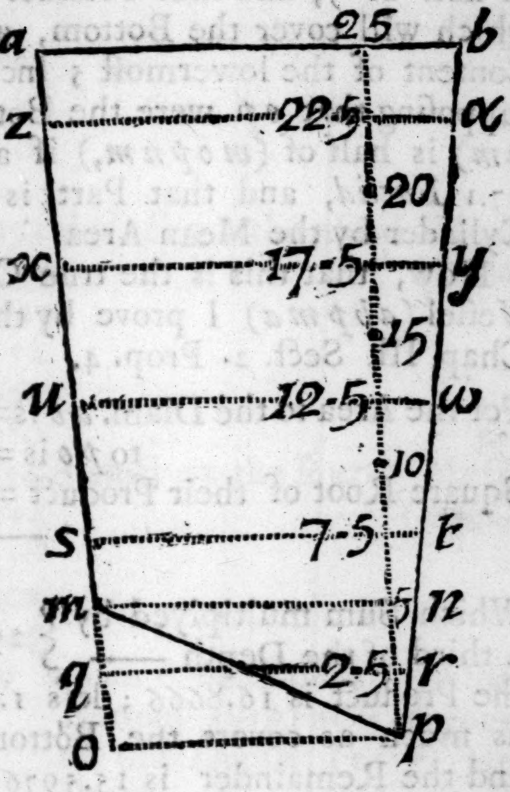
To find what ſome (improperly) call the Drip. or Fall of a Tun; for there is no Fall of the Tun, but of the Bottom: So I think it more intelligible and pertinent to ſay,

To find the Content and inch any Tun whoſe Bottom varieth, or hath a Reclination from a Parallel with the Surface of the Liquor, as m p *from* m n, s t, *&c.*

This having, in no Book of Gauging, been made plain, I ſhall endeavour here to do it by a Figure, and an Explanation, a preparatory Scheme, Rules, and an Example of inching the ſame.

a b p m a is a Tun; ſuppoſe they are fixed and their Poſition (by Law) not to be altered, ſo that for the more Conveniency of cleaning them, the Bottom is made to recline, as you ſee *m p* reclining from *m n*; and there is a Plug at *p* to take out, to let out the Waſh, and to be put in again. Here this Veſſel is the Fruſtum of a Cone, whoſe Depth 25 *p* is

=25 Inches, Diameter *ab* =18, and *po* =13 Inches: So that the Middle of every 5 Inch. deep is at 2.5, 7.5, &c. And the Diameters at thoſe depths, &c. are as in *this following preparatory Scheme.*



Middle of the	is at Inches deep.	Diameters at thoſe Depths.	Areas to thoſe Diameters.	Content of every 5 Inch. deep.
1ſt. 5 -	22.5 -	=17.5 - -	=.8529 - -	=4.2645
2d. 5 -	17.5 -	16.5 - -	.7582 - -	3.791
3d. 5 -	12.5 -	15.5 - -	.6691 - -	3.3455
4th. 5 -	7.5 -	14.5 - -	.5856 - -	2.928
5th. 5 -	2.5 -	13.5 - -	.5076 - -	1.269

Sum = the whole Content is =15.598 (Gal.

Here 'tis plain there is little of Difficulty as to the reclining Bottom; for I only multiply the Mean Area (ſuppoſe at *qr*) by 2.5, or half of 5, and that Product gives 1.269, which will cover the Bottom, and is half the Content of the lowermoſt 5 Inches of Depth, ſuppoſing that *op* were the Bottom; for (*mp nm*) is half of (*mopnm*,) if a Cylinder, by 37.1 *Euclid*, and that Part is reduced to a Cylinder by the Mean Area.

Now, that this is the true Content of this Veſſel (*abpma*) I prove by the Rule given Chap. III. Sect. 2. Prop. 4.

For the Area to the Diam. *ab* is=.9024
to *po* is =.4707
Square Root of their Product =.6517

2.0248=Sum.

Which Sum multiplyed by a third of the Depth —— } 25=8.33,
the Product is 16.8666; leſs 1.269 (which is as much as covers the Bottom of the Tun, and the Remainder is 15.5976, which is but .0004 of an Inch leſs than the Content, in the ſaid Scheme-Method. It is inched thus:

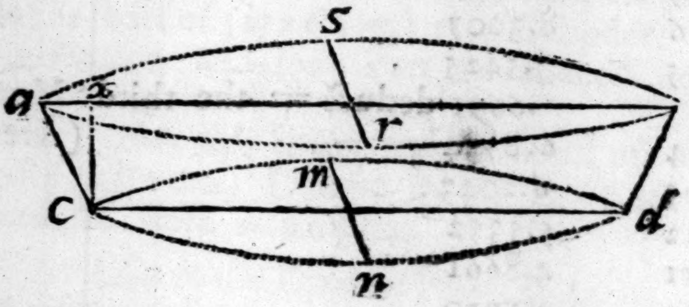
Depth Inches.	Content.
25	15.598
	.8529 deduct = the Mean Area (next the Top.
24	14.7451
23	13.8922
22	13.0393
21	12.1864
20	11.3335
	.7582 deduct = the ſecond Mean (Area.
19	10.5753
18	9.8171

Depth Inches.	Content.	
17	9.0589	
16	8.3007	
15	7.5425	
	.6691	deduct = the third Mean (Area.
14	6.8734	
13	6.2043	
12	5.5352	
11	4.8661	
10	4.1970	
	.5856	deduct = the fourth Mean (Area.
9	3.6114	
8	3.0258	
7	2.4402	
6	1.8546	
5	1.269	which Remainder covers the Bottom (*mp*) at 5 Inches of the Depth (25 *p*) for Proof.

Note once for all, That in Caſe the Surface of the Liquor is covered with Yeſt or Froth, a round, *&c.* Cork is put upon the End of the Rule, ſo that the End is even with the lower Side of the Cork; and that reſting on the Surface of the Liquor, the Inches upward are the dry Inches; which taken from the whole Depth, leaves the wet Inches, againſt which you have the Content as in the Tables.

Note alſo, That the Depth is always to be taken perpendicularly, as 25*p*, *&c.* and not by the Side as *bp*.

Prob. 6. *To gauge and inch a Cooler, which is a Conical Veſſel, but the Baſes are elliptical and Sides regular.*



Greater Diameter of the greater Baſe as *a b* —— } =144 Inches

Its Conjugate Diameter *r s* =109]

Greater Diameter of the leſſer Baſe as *c d* —— } =128

Its Conjugate Diameter *m n* = 97 Depth *x c* (= 15

What is the Content of this Cylindroidal Veſſel, and of every Inch?

The Preparatory Scheme or Table for inching the above Veſſel.

Middle of every 5 Inch. in Depth.	Means of the greater Diam.	Means of the leſſer Diam.	Squares of Diameters of Mean Circles.	Mean Areas in Gallons Beer.	Content of every 5 Inches deep in Gallons Beer.
12.5	141.3r	107	15122.6r1	42.1185	210.5925
7.5	136 --	103	14008.	39.014	195.07
2.5	130.6r	99	12735.9r1	36.0283	180.1415

Sum or Content of the Veſſel = 585.804

1. The firſt Column towards the Left-Hand, ſhews that the Middle of the firſt five Inches next the Top, between 10 and 15 deep, is 12.5; the Middle of the next 5 between 5 and 10 is 7.5; and between 1 and 5 is 2.5.

2. For the next Column, I firſt find the Means from Top to Bottom, againſt the ſaid Depths of the greater Diameters 144 and 128, the Difference of which being 16, to find the Means, I ſay,

As 15 : 16 :: 12.5 : 13.3*r*1; more 128 is 141.3*r* the firſt Mean, *&c.*

And for the Means of the leſſer Diameters 109 and 97, I ſay likewiſe,

As 15 : 12 :: 12.5 : 10; more 97 is 107, the firſt Mean: and ſo of the reſt.

3. The Products of the greater and leſſer Means, as of 141.3*r*, and 107, gives 15122.6*r*, *&c.*

4. Thoſe Squares of Diameters divided by 359.05 give the reſpective Mean Areas. And,

5. Thoſe Mean Areas, multiply'd by 5, produce the Contents in the fifth Column.

A Table of the Content of every Inch Deep of the laſt Veſſel.

Inches deep of Liquor.	Content at thoſe Inches deep in Gal. Beer or Ale.	Proof of the Work as you proceed.
15	585.804	
	42.1185 deduct	210.5925
14	543.6855	375.2115 Proof.
13	501.5670	
12	459.4485	

Inches deep of Liquor	Content at thoſe Inches deep in Gal. Beer or Ale.	Proof of the Work. as you proceed.
11	417.33	
10	375.2115	
	39.0140 deduct	
9	336.1975	
8	297.1835	195.07
7	258.1695	180.1415 Proof.
6	219.1555	
5	180.1415	
	36.0283 deduct	
4	144.1132	
3	108.0849	
2	72.0563	
1	36.0283	equal to the Sum deducted, which proves the whole.
0	0.	

Example 2. *To gauge and inch an Elliptical but Conical Tun.*

	Inches.
Whoſe Tranſverſe Diameters are, at the Top =	110
at the Bottom	100
Conjugate Diameters at the Top	96
at the Bottom	90

And the Depth is = 24 Inches.

1ſt, To find the Diameters in the Middle of every four Inches of Depth, I do thus: **Firſt**, for the mean Tranſverſe Diameters againſt 22, the Middle of the firſt four Inches next the Top of the Veſſel; I ſay, As 24, the whole Depth : to 10, the Difference of Diameters :: So is 22 : to 9.16*r*1. Which added to the Bottom Diameter gives 109.16*r*1 = the true Mean againſt 22. Or if it be deducted from the Diameter at the Top, it gives the Mean Tranſverſe Diameter againſt 2; and ſo of all the reſt of theſe, and of the Mean Conjugate Diameters.

2dly, For the Mean Areas to theſe Diameters; for Example, againſt 22 Inches deep: I multiply the two Means together, and divide the Product by 359.05 = the Square of the Gauge Point for an Ale or Beer Gallon. I will give an Example of the Operation, that the Reader may ſee how the repeating Digits are managed, agreeable to what is taught before in Decimals; a Way not much known.

```
  109.16r1
     95.5      359.05) 10425.416r1 (29.0361=
 ─────────                324441      the firſt
    54583r1               129666      Mean A-
   54583r1                 219516     rea, &c.
  98249r1                   40866
 ─────────
10425.416r1
```

3dly, Each Mean Area, multiply'd by 4, gives the Products next the Right-Hand; the Sum whereof is the Content of the Tun = the Sum of the Mean Areas multiplied by 4.

An *Example* of what is ſaid, is as follows.

Depth Inch.	Mean Tranſverſe Diameters.	Mean Conjugat Diameters.	Mean Areas in Gall.	Content of every 4 Inch. deep.
22	109.16r1	95.5	29.0361	116.1444
18	107.5	94.5	28.2934	113.1736
14	105.83r1	93.5	27.56	110.24
10	104.16r1	92.5	26.8358	107.3432
6	102.5	91.5	26.121	104.484
1	100.83r1	90.5	25.4155	101.662
		Sums Total =	163.2618	653.0472

4thly, From this preparatory Scheme I find the Content of every Inch deep of Liquid, as in the following Table, by deducting the firſt Mean Area from the whole Content, and again from the Remainder, *&c.* to 20: And the ſecond Mean Area = 28.2934, from the Content againſt 20 Inches deep, *&c.* down to 16, from the Content againſt which I deduct the third Mean Area, *&c.* And the laſt Content againſt 1, being = the laſt Mean Area, proves the whole to be duly calculated, which appears to be ſo in every reſpect: And

Every Inch deep contains thus.

Inches deep.	Ale or Beer Gallons.	Inches deep.	Ale or Beer Gallons.
24	653.0472	17	452.0226
23	624.0111	16	423.7292
22	594.975	15	396.1692
21	565.9389	14	368.6092
20	536.9028	13	341.0492
19	508.6094	12	313.4892
18	480.316	11	286.6534

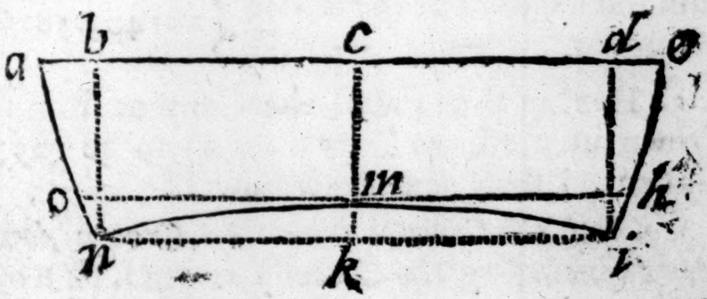
Inches deep.	Ale or Beer Gallons.	Inches deep.	Ale or Beer Gallons.
10	259.8176	5	127.783
9	232.9818	4	101.662
8	206.146	3	76.2465
7	180.025	2	50.831
6	153.904	1	25.4155

A general, but eaſy and uſeful Rule as to the Depth.

If the Depth be any Number of 10ths of Inches more than as in the Table: As ſuppoſe 15.6 deep; then I take the Difference between the Contents againſt 16 and 15, and multiply a 10th thereof by .6, and adding the Product = 16.536 to the Content againſt 15 = 396.1692, the Sum is the true Content for the Depth 15.6, *viz.* 412.7052, as by the Operation in the Margin.

Inch. deep. Content.
16———423.7292
15———396.1692
difference 27.56
a 10th of that 2.756
multiply by .6
Product add 16.536
15.6 = 412.7052

Prob. 7. *To find the Content of, and inch a Copper, and how to gauge and allow for the Crown.*



1. The neceſſary Dimenſions to be taken are

ace = the greateſt Diameter = 120
omb the Diameter at the Crown's Top = 95.4
nki the Diameter at the Bottom = 89.6
bn the greateſt Depth = 48.4
cm the Depth to the Crown = 40.
mk = *bn* leſs *cm* the Altitude of the Crown. ——— —— —— } = 8.4

2. We will give the preparatory Table for inching (and which exhibits the Content of the Copper) from the Crown upward, as *acebmoa*, viz.

The Preparatory Scheme or Table for inching the Copper

The Middle of every 8 Inch. deep.	Diameters at thoſe Depths.	Mean Areas to thoſe Diameters.	Content of every 8 Inch. Gal.
36	117.54	38.478	307.824
28	112.62	35.324	282.592
20	107.7	32.305	258.44
12	102.78	29.421	235.368
4	97.86	26.672	213.376

Sum = Content to the Crown = 1297.6
Gallons that cover the Crown = 105.981

Sum the Content of the whole Copper. — — } = 1403.581 Gal.

3. Having thus found the Content from the Crown upward, as (*acebmoa*) to be 1297.6 Gallons, I ſhall next ſhew how

To find the Content from the Crown downward (including the Crown) as **omhikno.**

The Diameter *o m h* 95.4
added to that *n k i* 89.6
the Sum is — — 185.
half of which is - - 92.5 Gallons.
whoſe Area is — — — 23.83
which multiply by the Depth *m k* = 8.4
gives the Content of *o m h i k n o* = 200.172

4. *For the Content of the Crown,* (which muſt be deducted from this laſt)

1. If it be taken as the Segment of a Sphere; the Diameter of the Segment *n k i* = 89.6, Altitude *k m* = 8.4: And by the Rule Chap. III. Prop. 7. of Sect. 2. the Altitude of the greater Segment is = 955.73; the Area of the Segment = 22.362; which being multiplied by the ſaid 8.4, gives 187.8408, which multiplied by half the Altitude of the greater Segment, more a 6th of that of the leſſer = 479.265, gives 90025.521; and that divided by the ſaid 955.73 gives
94.191, and there reſts ded. 94.191 = the
the Gallons that cover ——— Crown
the Crown — — = 105.981

5. This laſt Way I have ſhewn, becauſe it is certainly an accurate Method of finding the Content of the Crown; and therefore it may ſatisfy the Reader that the following Rule, giving ſo very near the ſame Quantity, is a genuine one, that may ſafely be uſed, and is very much leſs operoſe. *The Rule is,*

Multiply the Area 22.359 (which is that to the Diameter of the Baſe of the Crown *ik n*) by half the Altitude *mk* =4.2; and the Product is 93.9 = the Content of the Crown, as before, within 29 hundreds of a Gallon.

The aboveſaid Content that covers the Crown, being added to that of the Copper, from the Crown upward, is the Content of the whole Copper, as you ſee before 1403.581 Gallons. 'Tis inched thus.

Depth in Liq. above the Crown.	Content in Gallons Beer or Ale.	Depth in Liq. above the Crown.	Content in Gallons Beer or Ale.	
40	1297.6	20	577.964	
	38.478	19	545.659	
39	1259.122	18	513.354	
38	1220.644	17	481.049	
37	1182.166	16	448.744	
36	1143.688		29.421	
35	1105.21	15	419.323	
34	1066.732	14	389.902	
33	1028.254	13	360.481	
32	989.776	12	331.06	Gal. cover. the Crown 105.981
	35.324	11	301.639	
31	954.452	10	272.218	
30	919.128	9	242.797	
29	883.804	8	213.376	
28	848.48		26.672	
27	813.156	7	186.704	
26	777.832	6	160.032	
25	742.508	5	133.36	
24	707.184	4	106.688	
	32.305	3	80.016	
23	674.879	2	53.344	
22	642.574	1	26.672	
21	610.269			

The Uſe of this Table. Admit the Officer comes and finds in the Copper, 19 Inches deep of Liquor, from
the Top of the Crown, Gallons.
againſt that ſtands —— 545.659
To which add what covers the Crown - - - - 105.981 } add.

The Sum is the Liquor in the Copper } =651.64 Gal.

Done with abundance of Eaſe and Expedition, by this Table, which is made from the Preparatory Table preceding: The ſeveral Sums deducted, being the Numbers in the Column of Mean Areas, found to the ſeveral Diameters in the ſecond Column, which are taken at the ſeveral Depths in the Middle of every 8 Inches, as mentioned in the firſt Column toward the Left-Hand.

Sect. 2. *The Gauging of Caſks.*

I Have, in the laſt Section above, ſhew'd how to gauge and inch all manner of open Veſſels, in a Method, which I take to be much more ample and uſeful, than is any where exhibited: And I am now to ſhew the young Officer how to gauge Caſks, whether intirely full, or only in part ſo. Theſe Caſks are commonly taken for the Middle Fruſtum of a Spheroid, the Menſuration of which is ſhewed Chap. III. Sect. 2. Prop. 8. which Fruſtum is repreſented by the Figure *a b c n a*, Part of the Spheroid *o a b m c n o*, which is generated by the Rotation of the Semi-Ellipſis *o r s m h o* round its *Latus Tranſverſum o r s m.*

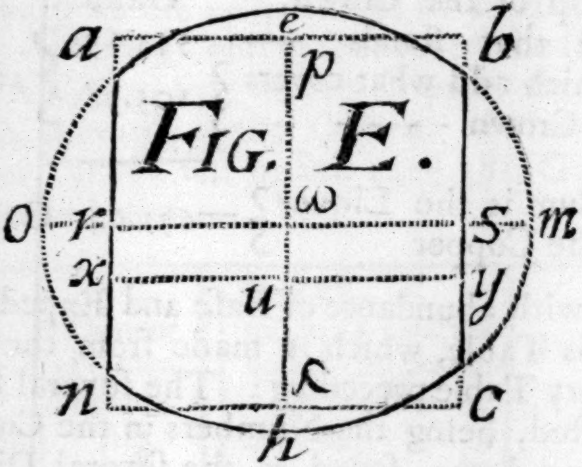


FIG. E.

Prob. 1. Admit the Bung-Diameter *e h* be Inches — — — } 40
Head Diameter *b c* = 28
Length of the Cask *r s* = 52

What is the Content of this Cask?

Rule 1. Multiply the Difference of the Diameters (12) by .7; add the Product 8.4 to the Head Diameter, gives 36.4, the Mean Diameter, whoſe Area 3.6907, multiplied by the Length 52, gives 191.91 the Content.

Rule 2. *Another Rule to do this, is,*

To the double Square of the Bung Diameter — — } = 3200
Add the Square of the Head Diam. = 784
Sum = 3984
Multiply that Sum by 52, the Cask's Length, and the Product is — } 207168
Which divide by 1077, and the Quote is Gallons. } 192.356

Which is more than the firſt Method by .446 of a Gallon.

But if inſtead of multiplying by .7 (as by the old Rule) you had done it by .70342, it would have agreed with the other Rules.

Rule 3. *Another Rule is,*

To the Sum and Half Sum of the Squares of the Diameters, add half the Difference of thoſe Squares; multiply that Sum by the Cask's Length, and divide the Product by 1077, gives the Anſwer.

Example. Diam. at the Bung 40, its Square —	1600
At the Head 28, its Square	784
Sum	= 2384
Half that Sum	= 1192
Difference of the Squares 816, half that	= 408
Sum	= 3984
Which multiplied by the Length 52, gives	207168
That divided by 1077 quotes Gallons	192.356 as before

Rule 4. *A fourth Rule to find the Quantity of Liquor in this Cask, is,*

To the double Square of the Diameter at the Bung, add the Square of the Diameter at the Head; multiply a third of that Sum by the Length of the Cask; and divide that Product by the common Diviſor or Square of the Gauge-Point 359, and the Quote is the Anſwer, as in the two laſt Rules.

Example.

Double Square of the Bung 3200
add the Sq. of the Head Diam. 784

Sum = 3984
a Third of that 1328
Multiply by 52

Which divide by 359) 69056 (192.356
and the Quotient is 192.356 3315
Gallons as before. 846
1280
2030
2450

Rule 5. *Another Way is* (for ſuch as would rather multiply than divide) To the double Square of the Bung add the Square of the Head, then multiply that Sum by .0009285, and that Product by the Length 52, and you have the Anſwer = 192.35548, as before to the .00052 of a Gallon, or $\frac{4}{1000}$ Part of a Pint.

6. But there are, it may be, ſome Sorts of Casks, which have leſs Curvature than the ſaid Middle Fruſtum of a Spheroid: And,

2dly, Others which have but little Curvature; and a

3d Kind which have no Curvature in the Staves, and are either the Middle Fruſtum of two Cones, having but one common Baſe, which is the Area to the Bung Diameter.

4thly, Or elſe a Cylinder whoſe Diameter is that of the Head of the Cask.

7. It is eaſy to get three Inſtruments, having different Degrees of Curvature, anſwering to the three Casks deſcribed above, by applying of which you may know the Curvature, and conſequently which of the following Rules to make uſe of.

1ſt, For the Spheroid, by the laſt Rules, the Content is 192.356 Gallons.

2d, For a Cask having leſs Curvature, of the ſame Dimenſions at the Bung, Head and Length; proceed to find the Content as under the 3d Rule above: but inſtead of adding half the Difference of the Squares, add three Tenths of the whole Difference, *viz.* 244.8, and multiply that Sum = 3820.8 by the Length, gives 198681.6, which divided by 1077 quotes 184.476 Gallons the Content.

3d, For a Cask of the ſame Dimenſions at the Bung, Head and Length, but having little Curvature; proceed as under Rule 3. above, but only, inſtead of Half, add one Tenth of the Difference of the Squares, and you'll find the Content 176.597 Gallons.

4th, If a Cask of the ſame Dimenſions at the Bung, Head and Length, have no Curvature of its Staves; the Rule to gauge this, is, from the Sum and Half Sum of the Squares of the Bung and Head Diameters (in theſe Dimenſions 3576) deduct half the Square of the Difference of the Diameters (72), the Remainder (3504), multiplied by the Length (52), and the Product being divided by 1077, gives the Content (169.181 Gallons.)

8. By the Operations above, it appears that a Cask of the Dimenſions, 40 Bung Diameter,

28 = Head Diameter, and 52 Inches long, will contain,

	Gallons.
If it be the Middle Fruſtum of a Spheroid,	192.356
If ſomewhat leſs in Curvature —	184.476
If the Staves have little Curvature =	176.597
If no Curvature, it would contain =	169.181
If the Cask were a Cylinder of the ſame Length (52), and Diameter that of the Head, which in the Example above is 28 Inches,	113.56

Which Contents will be found very neceſſary to be exhibited, when we come to ſhew how to find the Ullage of a Cask that lieth in the Poſition of that above, with its Bung Diameter, &c. perpendicular to the Horizon.

9. Becauſe in the next, &c. Rules, the Words, verſed Sine and Segment are uſed, I ſhall therefore explain the Lines in and Parts of a Circle.

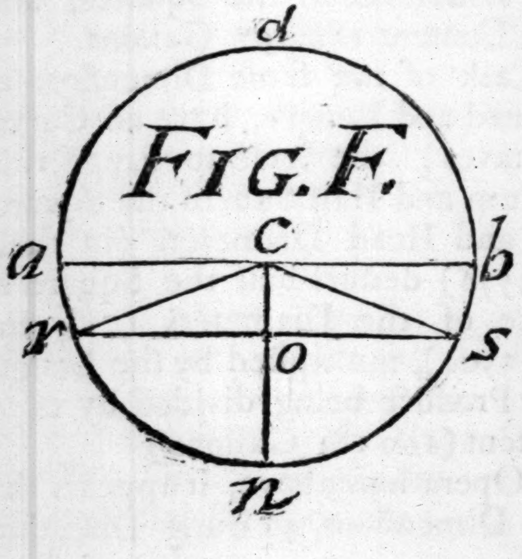


FIG. F.

1ſt, For the Lines; *a b* is the Diameter, *a c* = the Semi-Diameter or Radius, *a d b n a* the Periphery, Ambit, or Circumference; *r s* a Chord Line; (*o n*) a verſed Sine.

2d, For the Parts; *a d b a* =

=the Semi-Circle, *a c n a* a Quadrant or 4th of a Circle, *r c s n r* a Sector of the Circle, *r o s n r* a Segment of the Circle. Theſe are all that are neceſſary to be mentioned here.

A Table follows to help us to find the Quantity of Liquor in a Cask (as the laſt above) when Part is drawn out of it, or the Ullage, or what it wants of being full.

The following Table, for its great Uſe (which, and its Calculation I have ſhewn) hath been inſerted in all Books of Gauging, for theſe 50 Years paſt, and 'tis ſaid to have been the Contrivance of one Monſieur *Hans*.

A Table of the Areas of Segments of a Circle, whoſe Circumference is 3·5449077, *Area* =1, *and Diameter* = 1.1283792, *divided into* 100 *equal Parts.* Carefully examined and corrected *per E. H.*

Verſed Sines.	Areas of Segments.	Verſed Sines.	Areas of Segments.
1	.0017	16	.1033
2	.0048	17	.1127
3	.0087	18	.1224
4	.0134	19	.1323
5	.0187	20	.1424
6	.0245	21	.1527
7	.0308	22	.1631
8	.0375	23	.1737
9	.0446	24	.1845
10	.0520	25	.1955
11	.0598	26	.2066
12	.0680	27	.2178
13	.0764	28	.2292
14	.0851	29	.2407
15	.0941	30	.2523

31

Verſed Sines.	Areas of Segments.	Verſed Sines.	Areas of Segments.
31	.2640	66	.7002
32	.2759	67	.7122
33	.2878	68	.7241
34	.2998	69	.7360
35	.3119	70	.7477
36	.3241	71	.7593
37	.3364	72	.7708
38	.3487	73	.7822
39	.3611	74	.7934
40	.3735	75	.8045
41	.3860	76	.8155
42	.3986	77	.8263
43	.4112	78	.8369
44	.4238	79	.8473
45	.4364	80	.8576
46	.4491	81	.8677
47	.4618	82	.8776
48	.4745	83	.8873
49	.4873	84	.8967
50	.5000	85	.9059
51	.5127	86	.9149
52	.5255	87	.9236
53	.5382	88	.9320
54	.5509	89	.9402
55	.5636	90	.9480
56	.5762	91	.9554
57	.5888	92	.9625
58	.6014	93	.9692
59	.6140	94	.9755
60	.6265	95	.9813
61	.6389	96	.9866
62	.6513	97	.9913
63	.6636	98	.9952
64	.6759	99	.9983
65	.6881		

Prob. 2. *The Uſe of this Table, to find the Ullage of a Cask, or Liquor remaining.*

Admit I come to the Cask above, and find it 15 Inches wet, (as that the Liquor comes up to the Line *xy*, Fig. *E.*) to know what Number of Gallons are contained in the Cask, this is the Rule.

1ſt, I ſay, as 40, the Bung Diameter, is : to 15, the wet Inches :: So is 1 : to 37½, the verſed Sine (whoſe Area of Segment, by the Table is 3425.)

2dly, As 1 : to 192.356, the Content :: So is 3425 : to 65.882 Gallons in the Cask, being the Fruſtum of a Spheroid.

3dly, And the wet Inches being the ſame, the verſed Sine and Area of the Segment are the ſame for the Casks of leſs Curvature : Therefore,

As 1 : to 184.476 :: So is the Segment .3425 : to 63.183 Gallons.

4thly, As 1 : to 176.597 :: So is 3425 : to 60.484 Gallons.

5thly, And as 1 : to 169.181 :: So 3425 : to 57.944 Gallons ; ſo that the wet Inches, being 15 in each Cask, there remains,

	Gallons.
In the Spheroid's middle Fruſtum	65.882
In the Cask of leſs Curvature	63.183
In the Cask of little Curvature	60.484
And in the Cask of no Curvature	57.949

And if you would know the Ullage of each Cask ; divide the dry Inches by the Bung Diameter, gives the verſed Sine ; the Area of the Segment to which, multiplied by the Content of the Cask, gives the Ullage, or what ſuch Cask wants of being full.

As ſuppoſe, in the Middle Fruſtum of a Spheroid, I find 25 dry Inches; that divided by the Bung Diameter, gives 62½; the Area of the Segment to which is = .6574, which multiplied by the Content 192.356, produceth Gallons 126.474, the Ullage or what the Cask wants of being full.

But Note, That the verſed Sine being 62 and a half, I therefore add half the Difference between .6513 (the verſed Sine of 62) and 6636 (the verſed Sine of 63) which half Difference 61, more 6513, gives .6574, which I take for the verſed Sine: And this is always to be minded, or elſe your Calculations will fall ſhort.

10. *A double Proof that this Way of finding the Ullage and what remains in a Cask, if it be the Middle Fruſtum of Spheroid, is true; tho' it has been denied by ſome Men of ſome Note.*

1ſt Way ſhall be by the Mean Diameter of the Spheroid, which is ſound by multiplying the Differerence of the Diameters by .70342, and adding the Product to the Head Diameter, gives 36.44104 = the Mean Diameter.

I come and find at the Bung Diameter 15 wet Inches; which Bung Diameter being 3.559 more than the ſaid Mean: I ſay, if 40, the Bung Diameter, do abate 3.559, what muſt the 15 wet Inches abate? Anſwer, 1.3346, which therefore deduct from the 15, and there reſts 13.6654, which I divide by the

Mean Diameter 36.441, and the Quote is 37½ the verſed Sine, as in the common Way; whoſe Segment, as taught above, being .3425, if multiplied in the Content . 192.356, gives 65.882, the Liquor remaining in the Caſk; the very ſame as by the Method uſually practiſed.

2. A ſecond Rule to prove the Truth of the Way, always uſed in finding the Ullage and Liquor remaining in a Caſk, is in the Examples of the Uſe of the Table above, where the Liquor found in the Middle Fruſtum of a Spheroid, being upon

15 wet Inches —— —— 65.882 wet

And upon the Complement thereof, *i. e.* 25 Inches dry, the Ullage being — } =126.474 dry

TheSum is theContent of the whole 192.356 as is manifeſt by the four firſt Rules of Caſk-gauging.

3. *A third Way to find the Ullage of a Caſk, or (in the Example following) the Quantity of Liquor remaining therein, after a new Method, (as the former Way laſt above is) but without the Table of Segments; which will alſo be a Rule how to calculate that Table.*

Suppoſe the ſame Caſk and Dimenſions as before,

Fig. *E.* *above.* Bung Diameter - = 40 = *e h*
Head Diameter - = 28 = *b c*
Length of the Caſk = 52 = *r s*
Wet Inches of the Bung Diameter = 15 = *u k*

Mean Diamet. found as above $36.441 = p$ λ
Wet Inches of the Mean Diameter, by the Rule above } $13.6654 = u$ λ
Half the Mean Diameter - - $18.2205 = \omega$ λ

1. I take ½ the Mean Diameter from a Diagonal Scale, and with that Extent of the Compaſſes, deſcribe a Circle (ſuppoſe *a b d n r* in Fig. *F.* above) which repreſents the End of the Cask, when reduced to a Cylinder.

2. From the ſame Diagonal Scale I take 13.66, and ſet from *n* to *o*.

3. Thro' *o* I draw a Line parallel to the Diameter *a b*, as the Chord Line *r s*; and where that Line cuts the Ambit, I draw *r c* and *c s*.

4. With a Line of Chords, &c. I take the Angle at *c*, viz. *r c s*, which I find 150.5 Degrees, which is the Arc of the Sector, as *r n s c r*.

5. The next Thing, *in order to find the Area of the Sector* r c s n r, is to convert the 150.5 Degrees into Inches; in order to which I ſee what the Circumference to the Mean Diameter 36.441 is, which I find 114.9146. Then I ſay,

As 360, the Degrees in a Circle, is : to 114.9146, the Inches of my Circumference, as aboveſaid :: So is 150.5, the ſaid Angle of the Sector : to 48.0406 Inches = the ſaid Arc *r n s*.

6. I multiply a 4th of that, = 12.01015, by the Mean Diameter 36.441, and the Product is 437.6618 Inches in the Sector.

Then for the Content of the Triangle in the Sector, as rcsor, *which muſt be deducted:*

7. By the 47.1 *Euclid*, the Square of half the Mean Diameter, 18.22 = cr; = 331.9684, leſs the Square of co = 4.555, (which is cn leſs (on) or the Semi-Diameter, leſs the wet Inches of the Mean Circle) which Square is 20.748, there reſteth 311.22; whoſe Square Root is 17.64 = the Semi-Chord ro.

8. This 17.64, multiplied by 4.555 = co, produceth 80.3502 = the Area of the Triangle; which deducted from the Area of the Sector = 437.66187, there reſteth 357.31167 = the Area of the Segment $rosnr$ in Inches, which divide by 282, gives 1.26706 = the Area in Gallons.

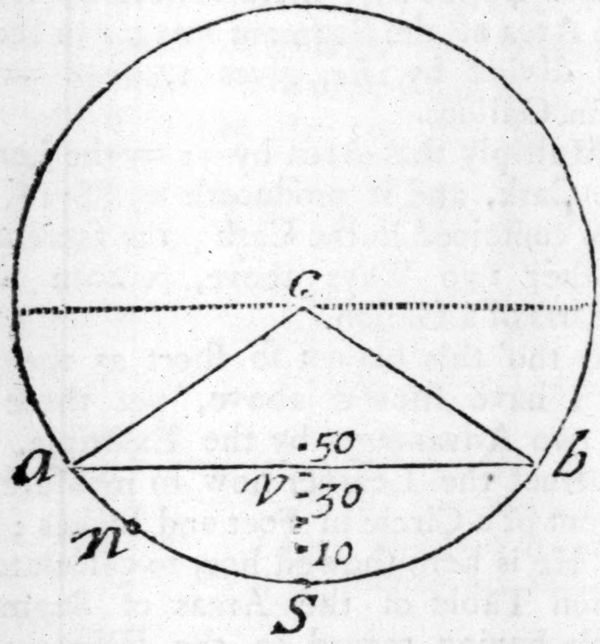
9. Multiply that Area by 52 = the Length of the Cask, and it produceth 65.88712, the Liquor contained in the Cask; the ſame as by the other two Ways above, within about .005 Parts of a Gallon.

And tho' this be not ſo ſhort as the Methods I have ſhew'd above, yet there are theſe two Advantages by the Example. 1ſt, To inſtruct the Learner how to meaſure the Segment of a Circle in Feet and Inches; and, 2dly, He is here ſhewed how to calculate the common Table of the Areas of Segments, by only having regard to the Difference in Dimenſions.

And thus I have given the Reader a greater Variety of Rules concerning this moſt difficult Part of gauging a Cask and its Parts, than is elſewhere publiſhed; and ſhall alſo add ſomething new, as to Casks ſtanding on one End, by and by.

4. *But the moſt eaſy Rule to find a tabular Number in the foregoing Table, or Area of a Segment to any verſed Sine, is this.*

1. Conſider that the Areas of Sectors and the Triangles therein, and conſequently the Segment, are the eaſieſt found from the Circle, whoſe Diameter is 2, which I ſhall therefore make uſe of; and ſhall, ſecondly, ſhew how the Segments of that are reduced to thoſe in the Table.



2. Next Thing is, to take 60 Degrees between your Compaſſes, from a good Line of Chords (the larger the Radius is, the better) and deſcribe the Circle as you ſee here.

3. Then divide the Radius *c s* into 100 Parts, and ſet off 40 (ſuppoſe the verſed Sine whoſe Segment you would know) from *s* to *v*, and draw the Chord Line *a u b*. Then with 90 Degrees between your Compaſſes, ſet one Foot in *b*, and the other will extend to *n*; and taking *a n* between your Compaſſes, applying that to your Line of Chords, you'll find it 16 Degrees 18 Min. which added to 90, gives $106^\circ\ 18' =$ the Angle *a c b*, or at *c*; Half of which is $53^\circ\ 9'$, the Angle *a c s*, or Half the Angle at *c*.

4. The next Thing is, to convert this Arc *a n s* into Inches, and (the 9′ being .15 of a Degree) to get thoſe Inches, I ſay,

As 360, the Degrees in the whole Circle is : to 6.2831852, the whole Circumference in Inches :: So is this Arc $a\,s = 53.15^\circ$: to .927642895 Inches in *a s*, which is alſo the Area of the Sector *a c b s a*; from the Rules given before for meaſuring the Parts of a Circle, Chap. III. Sect. 1. Prop. 10.

The next Steps are, to find the Area of the Triangle *a c b a*; to deduct it, in order to which,

5. I ſay, the Semi-Diameter $c\,s = 1$, leſs the verſed Sine $v\,s = 40$ Parts, leaves the Complement $= c\,v = 60$ Parts.

6. By the 47.1 *Euclid*, the Square of 1 $= a\,c$, leſs the Square of $v\,c = 0.3600$, leaves .6400, whoſe Square Root .80 is the Semi-Chord *a v*.

7. Multiply that .80 by $c\,v = 60$, gives .4800 the Area of the Triangle *a c b a*.

8. Deduct that from the Area of the Sector $= .927642895$, and the Remainder is the

Area of the Segment $= absa =$ .447642895.

9. Now having found the Segment of a Circle whoſe Diameter is 2; to find the Area of that of the ſame Segment in the Table, ſay,

As 3.1415926 = the Area of a Circle whoſe Diameter is 2, is : to .447642895, the Area of the Segment found :: So is 1, the Area of the Circle for which the Table of Segments is made : to .1424, the true Segment in the Table, anſwering to 20.

10. For as in the former Circle, from whence the Calculations were made, the verſed Sine was 40; ſo in the ſecond, or that whence the Tables are made, it is but 20; becauſe Radius in the former Circle was divided into 100 equal Parts, 40 of which was the verſed Sine propoſed: So in Proportion the Radius of the ſecond Circle, being but divided into 50 Parts, the verſed Sine muſt be but 20. Thus,

As 100 : 40 :: 50 : 20.

And whatſoever verſed Sine in the Table you would find the Segment to, you muſt double it to make your Calculations from, in the previous Circle, whoſe Diameter is 2 or 200 equal Parts, and Radius 100; 40 being 2 and a half of the Radius 100, as 20 is 2 and a half of the Radius 50. And if you'd find the Segment of 15 in the Table, you muſt do it firſt by the verſed Sine 30, of the Circle whoſe Diameter is 2; the former of which being .3*r* Parts of the Radius 50, as the laſt mentioned 30 is .3*r* Parts of the

Radius 100; and the like may be ſaid of any others.

Prob. 3. *Of Cask-gauging, ſhall be to find the Liquor in a Cask (or the Ullage when it is only in part filled) which ſtands on one End, with its Diameters parallel, and its Axis or Length perpendicular to the Horizon*; as the Figure following.

Suppoſe this Cask's Bung Diameter } = 22 Inches = *o e*
Head Diameter = 17 = *a c*
Length —— = 32 = *b m*

1. There have been five Ways exhibited above, to find the whole Content hereof, by any of which it will be found to be 37.348 Gallons; ſo half the Content is 18.674, and half the Length = 16; the Surface of the Liquor in the Cask, being *r d*, diſtant 10 Inches above the Bung Diameter *o e*, or the Line *x t*; how much is in the Cask, and what does it want of being full ?

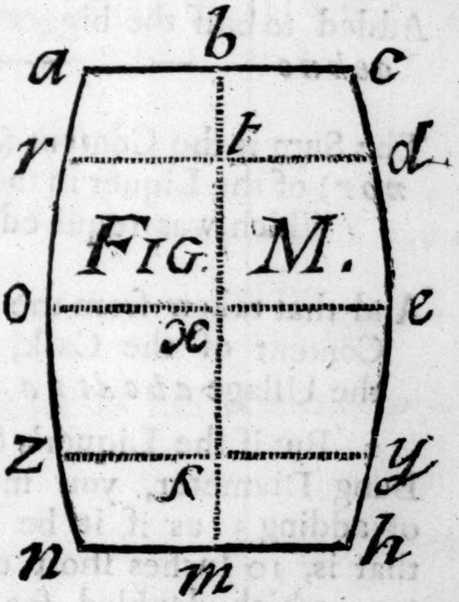


FIG. M.

Rule. Suppoſe a middle Segment within this Cask, whoſe Length is as much below the *x*, as *t* is above it, you will then have the Cask's Segment *rtdeyzor*, within the Cask given *abcehnoa*; ſo that we have nothing to do but find the Content of the Middle Fruſtum of the Cask given, (or of the inner or leſſer Cask) by the common Rule given for the Middle Fruſtum of a Spheroid; and adding half that, to half the Content of the greater Cask, gives the Anſwer: Which Rule is not only new, but moſt eaſy and natural.

Examp. By what is above, the Bung Diameter of this inner Cask, or rather Middle Fruſtum of the Cask given, as (*o e*) is 22 Inches, length $t\lambda = 20$; and the Head Diameter is found (by the Rules to be given below) $= rtd = 20.194$.

Gallons.

Upon theſe Dimenſions the Content of the inner Cask or Fruſtum *rdyzr* is — — 25.549

Half of that is *rdexo* = 12.774

Added to half the bigger Cask *oehno* — — = 18.674

The Sum is the Content (*rdehnor*) of the Liquor in the Cask Which was required. = 31.448

37.348

And that taken from the whole Content of the Cask, gives the Ullage *abcdtra* = = 5.9

2. But if the Liquor's Surface is below the Bung Diameter, you muſt ſubſtract inſtead of adding; as if it be λm 6 Inches deep, that is, 10 Inches ſhort of the Bung Diameter; which doubled for the Length of the

inner or leſſer Cask, is 20, and the Diameter $z\lambda y$ (or that at the like Diſtance above the Bung Diameter as (rtd) being = 20.194, the Content of that Cask, as is above ſhewed,

is Gallons — —	25.549
Half of which is —	12.774
Which taken from half the whole Cask — —	= 18.674
There reſts the Liquor in the Cask $z\lambda yhnz$	= 5.9 or *6 prope*

And thus may any Content in Liquor or Ullage be found for any Depth of Liquor above or below the Bung Diameter, by any one who only underſtands the Gauging of the whole Cask, or Middle Fruſtum of a Spheroid.

3. *To find any of the Diameters between the Bung and the Head.*

The Rule is (to inſtance, for the Diameter, 10 Inches from that of the Bung, which is made uſe of in the Example above.)

From the Square of the Bung Diameter —	22	= 484
Take the Square of the Head Diameter	17	= 289
		195 reſts.

And the Square Root of the Remainder is 13.964.

That Root divided by 16 (half the Length of the Cask) gives	.8728
Which multiplied by 10 (the Diſtance of the required Diameter from the Bung) gives	8.728
The Square of which, *viz.* —	76.177984
Deducted from the ſaid Sq. of the Bung Diameter	= 484
The Remainder is	= 407.822016

And is the Square of the Diameter required, whoſe Root is = 20.194: And ſo of any other Diameter.

4. But this may be done much eaſier, to the hundredth Part of an Inch, by a Diagonal Scale, and a large Pair of fine pointed Dividers, and laying the Dimenſions down upon Paper. And,

5. This Rule will come near enough the Truth, where there is little or no Curvature of the Staves; *i. e.*

As 16, half the Length of the Cask, is : to 5, the Difference of the Bung and Head Diameters :: So is 10 (the Diſtance of the Diameter required from the Bung) : to 3.125, which added to the Head Diameter, gives 20.125, which is but .069 of an Inch too little, even for the Fruſtum of a Spheroid.

6. *I ſhall next ſhew the inching of this Cask*, and conclude my Gauging of Ale and Beer.

I have inſerted the whole Work at large, and you will eaſily obſerve that four Mean Diameters, one in the Middle of every 4 Inches, are taken for one half or 16 Inches of the

Depth; and they alſo ſerve for the lower half of the Cask: And that the Areas to thoſe Diameters are deducted, firſt, the leaſt, &c. for the upper Part, and then the greateſt, &c. for the lower Part; all which is done in an eaſy and accurate Method. Thus,

The Preparatory Scheme or Table for inching this Cask.

Inch. from the Top.	Inch. from the Bung.	Square of the Diameters at thoſe Diſtances.	Mean Diam. Inches.	Mean Areas in Gal.	Content of every 4 Inches deep Gal.
2	14	334.691151	18.294	.9322	3.7288
6	10	407.822016	20.194	1.1359	4.5436
10	6	456.575925	21.367	1.2713	5.0852
14	2	480.952880	21.93	1.3292	5.3168
		Sum or half the Cask's Content =			18.6744

Here follows the Content of every Inch deep of the ſaid laſt Cask.

	At Inch. deep.	Content.	At Inch. deep.	Content.	
	32	37.3488	23	27.8051	
deduct—	—	.9322	22	26.5338	
	31	36.4166	21	25.2625	
	30	35.4844	20	23.9912	
	29	34.5522		1.3292	deduct
	28	33.62	19	22.662	
deduct—	—	1.1359	18	21.3328	
	27	32.4841	17	20.0036	
	26	31.4482	16	18.6744	= half
	25	30.2123	the Content of the Cask		
	24	29.0764	reſt at ' the Depth.		
deduct—	—	1.2713		1.3292	deduct.

reſts as above, againſt 15 deep, &c.

At Inch. deep.	Content.
15	17.3452
14	16.016
13	14.6868
12	13.3576
deduct—	— 1.2713
11	12.0863
10	10.815
9	9.5437
8	8.2724
deduct—	— 1.1359
7	7.1365
6	6.0006
5	4.8647
4	3.7288
	.9322 deduct
3	2.7966
2.	1.8644
1	.9322 = the laſt Subducend for Proof.
0	.0

Sect. 3. *Concerning Wine Casks, &c. at the Port of* London, *and the Gauging Wine.*

1. BUT firſt I ſhall give this Example from the Table of the Number of Wine-Gallons in any Number of ſolid Inches to 100 Gallons, under Sect. 5. Chap. 4. foregoing, which will ſhew its Uſe.

Suppoſe a Caſk of Wine, whoſe Bung Diameter is = 22, Head Diameter 17, Length 32, what does this Caſk contain? Anſwer, 45 Gallons, 3 Quarts *ferè*.

To know which, multiply .703 by the Difference of the Diameters 5, and the Product is 3.515; which added to the leſſer Diameter is = 20.5; that ſquared is 420.25; which multiplied by .7854, produceth 330.065; and that multiplied by the Length of the Caſk, is 10562 Inches.

And by the ſaid Table againſt the next leſs is 10395 (is 45 Gallons) which deducted, leaves 167 Inches, which by Table 1 of the Denominations of Wine-Meaſure, is near 3 Quarts. And this may be called a 6th Way of gauging the Fruſtum of the Spheroid; but though it be plain and eaſy, I cannot recommend it for Brevity.

2. The Casks that come here have either pretty much Curvature, as the *Rheniſh* Wine Fat, the *Canary* Pipe, *Spaniſh* Butt, *&c.* Or,

Thoſe whoſe Staves have leſs Curvature, as the *French* Wine Hogſhead, the *Florence* Wine-Casks, *&c.* all which may be meaſured by the Rules above delivered, and I ſhall only take Notice here of a 7th Way by which Casks have been ſometimes gauged; and that is by,

The Diagonal Line.

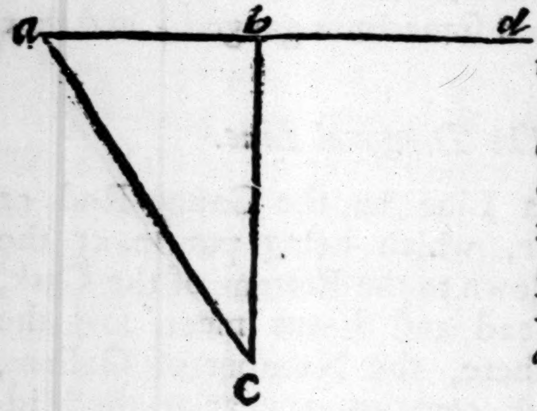
3. This is a Line on the Gauge-Rod or four-foot Rule, which being put in at the Bung-Hole, down to the Bottom of the Cask, where the Head and Staves meet, and the End reſting there, the Number of Gallons, which the Cask contains, appear at the Middle of the Bung-Hole, even with the inſide of the Staves. But if you ſuſpect the Bung-Hole is not juſt in the Middle of the Cask, you muſt put down your Rod to the Middle of the Bottom of both Ends, and add the Gallons that appear by each, and take half for the Content.

4. If your Cask is actually or very near a Cylinder, then the 47.1 *Euclid*, may be applied in finding the Diameter, Length of the Cask and Diagonal Inches, by having any two given.

Examp. Having the Diameter $= 30 = cb$, Diagonal $= 50 = ac$, to find the Length of the Cask $= da$.

Rule. From the Square of the Diagonal $=$ 2500, take the Square of the Diameter $=$ 900, the reſt is $= 1600$, whoſe Square Root is $= 40 = ab =$ half the Length, which doubled is 80, $=$ the Length of the Cask.

Examp. 2. The Diameter $cb = 30$, the Length $ad = 80$, to find the Diagonal ac.



Rule Extract the Squ. Root of the Squares of half $a\,d$ and of $cb = 1600$, more $900 =$ 2500, which Root is $50 =$ ac.

Exam. 3. The Diagonal $ac =$ 50, the Length $ad = 80$, to find bc the Diameter.

Rule. From the Square of $ac = 2500$, take the Square of half the Length, or $ab =$ 1600, and the Remainder is $= 900$, whoſe Square Root is $30 = cb$, the Diameter required.

5. They have a Rule for Oil, to allow 18 ℔. *per* 112 for Tare, and then they reduce the nett Weight into Liquid Meaſure, by allowing 7.5 ℔. to the Gallon: As ſuppoſe the Nett-Weight were ℔. 5250, that being divided by $7\frac{1}{2}$, gives 700 Gallons, or 5 Pipes, 1 Hogſhead and 7 Gallons, or 20 Jarrs, each 35 Gallons.

6. For a cloſe Cask they allow for the Thickneſs of the Head and Staves thus.

	Heads.	Staves.
For Pipes and Puncheons	1	.75 Inches
And for Hogſheads	75	.5

Table I. *Of Areas of Circles in Ale or Beer Gall.*

Parts of an Inch.

Diameters		.1	.2	.3	.4
Inch.	Areas.	Areas.	Areas.	Areas.	Areas.
10	0.2785	0.2841	0.2897	0.2954	0.3012
11	0.3370	0.3432	0.3494	0.3556	0.3620
12	0.4011	0.4078	0.4145	0.4214	0.4282
13	0.4707	0.4780	0.4853	0.4927	0.5001
14	0.5459	0.5537	0.5616	0.5695	0.5775
15	0.6266	0.6350	0.6435	0.6520	0.6605
16	0.7130	0.7219	0.7309	0.7400	0.7491
17	0.8049	0.8144	0.8239	0.8336	0.8432
18	0.9024	0.9124	0.9225	0.9327	0.9429
19	1.0054	1.0160	1.0267	1.0374	1.0482
20	1.1140	1.1252	1.1364	1.1477	1.1590
21	1.2282	1.2400	1.2517	1.2636	1.2755
22	1.3480	1.3603	1.3726	1.3850	1.3975
23	1.4733	1.4862	1.4991	1.5120	1.5250
24	1.6042	1.6176	1.6311	1.6446	1.6581
25	1.7407	1.7546	1.7686	1.7827	1.7968
26	1.8827	1.8972	1.9118	1.9264	1.9411
27	2.0303	2.0454	2.0605	2.0757	2.0909
28	2.1835	2.1991	2.2148	2.2306	2.2464
29	2.3423	2.3585	2.3747	2.3910	2.4073
30	2.5066	2.5233	2.5401	2.5570	2.5739
31	2.6765	2.6938	2.7111	2.7285	2.7460
32	2.8519	2.8698	2.8877	2.9057	2.9237
33	3.0330	3.0514	3.0698	3.0884	3.1069
34	3.2196	3.2385	3.2576	3.2766	3.2958
35	3.4117	3.4313	3.4509	3.4705	3.4902
36	3.6095	3.6296	3.6497	3.6699	3.6901
37	3.8128	3.8334	3.8541	3.8749	3.8957
38	4.0217	4.0429	4.0641	4.0854	4.1068
39	4.2361	4.2579	4.2797	4.3016	4.3235
40	4.4562	4.4785	4.5008	4.5233	4.5457

Parts of an Inch.

Diam.	.5	.6	.7	.8	.9
Inch.	Areas.	Areas.	Areas.	Areas.	Areas.
10	0.3071	0.3129	0.3189	0.3249	0.3309
11	0.3683	0.3748	0.3813	0.3878	0.3944
12	0.4352	0.4422	0.4492	0.4563	0.4635
13	0.5076	0.5151	0.5227	0.5304	0.5381
14	0.5856	0.5937	0.6018	0.61	0.6183
15	0.6691	0.6778	0.6865	0.6953	0.7041
16	0.7582	0.7675	0.7767	0.7861	0.7955
17	0.8529	0.8627	0.8725	0.8824	0.8924
18	0.9532	0.9635	0.9739	0.9843	0.9949
19	1.0590	1.0699	1.0808	1.0919	1.1029
20	1.1704	1.1819	1.1934	1.2049	1.2166
21	1.2874	1.2994	1.3115	1.3236	1.3358
22	1.4100	1.4225	1.4351	1.4478	1.4605
23	1.5381	1.5512	1.5644	1.5776	1.5909
24	1.6718	1.6854	1.6992	1.7129	1.7268
25	1.8110	1.8252	1.8395	1.8539	1.8683
26	1.9558	1.9706	1.9855	2.0004	2.0153
27	2.1062	2.1216	2.1370	2.1524	2.1679
28	2.2622	2.2781	2.2941	2.3101	2.3261
29	2.4237	2.4402	2.4567	2.4733	2.4899
30	2.5908	2.6079	2.6249	2.6421	2.6592
31	2.7635	2.7811	2.7987	2.8164	2.8341
32	2.9418	2.9599	2.9781	2.9963	3.0146
33	3.1256	3.1443	3.1630	3.1818	3.2007
34	3.3150	3.3342	3.3535	3.3729	3.3923
35	3.5099	3.5297	3.5496	3.5695	3.5895
36	3.7104	3.7308	3.7512	3.7717	3.7922
37	3.9165	3.9375	3.9584	3.9795	4.0005
38	4.1282	4.1497	4.1712	4.1928	4.2144
39	4.3455	4.3675	4.3896	4.4117	4.4339
40	4.5683	4.5908	4.6135	4.6362	4.6589

Parts of an Inch.

Diam.	.0	.1	.2	.3	.4
Inch.	Areas.	Areas.	Areas.	Areas.	Areas.
41	4.6818	4.7046	4.7275	4.7505	4.7735
42	4.9129	4.9363	4.9598	4.9834	5.0069
43	5.1496	5.1736	5.1977	5.2218	5.2459
44	5.3920	5.4165	5.4411	5.4657	5.4904
45	5.6398	5.6649	5.6901	5.7153	5.7405
46	5.8933	5.9189	5.9446	5.9704	5.9962
47	6.1523	6.1785	6.2048	6.2311	6.2575
48	6.4169	6.4436	6.4705	6.4973	6.5243
49	6.6870	6.7143	6.7417	6.7692	6.7966
50	6.9628	6.9906	7.0186	7.0466	7.0746
51	7.2440	7.2725	7.3010	7.3295	7.3581
52	7.5309	7.5599	7.5890	7.6181	7.6472
53	7.8233	7.8529	7.8825	7.9122	7.9419
54	8.1214	8.1515	8.1816	8.2118	8.2421
55	8.4249	8.4556	8.4863	8.5171	8.5479
56	8.7341	8.7653	8.7966	8.8279	8.8593
57	9.0489	9.0809	9.1124	9.1443	9.1762
58	9.3691	9.4014	9.4338	9.4662	9.4988
59	9.6949	9.7278	9.7608	9.7938	9.8268
60	10.0264	10.0598	10.0933	10.1269	10.1605
61	10.3634	10.3974	10.4314	10.4655	10.4997
62	10.7059	10.7405	10.7752	10.8098	10.8446
63	11.0541	11.0892	11.1244	11.1597	11.1953
64	11.4078	11.4435	11.4792	11.5150	11.5509
65	11.7670	11.8033	11.8396	11.8760	11.9124
66	12.1319	12.1688	12.2056	12.2425	12.2795
67	12.5023	12.5397	12.5771	12.6145	12.6520
68	12.8783	12.9162	12.9542	12.9922	13.0303
69	13.2599	13.2983	13.3368	13.3754	13.4140
70	13.6470	13.6860	13.7251	13.7642	13.8034
71	14.0397	14.0793	14.1189	14.1586	14.1983
72	14.4380	14.4781	14.5183	14.5585	14.5988

Parts of an Inch.

Diam.	.5	.6	.7	.8	.9
Inch.	Areas.	Areas.	Areas.	Areas.	Areas.
41	4.7966	4.8198	4.8430	4.8662	4.8895
42	5.0306	5.0543	5.0780	5.1019	5.1257
43	5.2701	5.2944	5.3187	5.3430	5.3675
44	5.5152	5.5400	5.5649	5.5898	5.6148
45	5.7659	5.7912	5.8167	5.8421	5.8677
46	6.0221	6.0480	6.0740	6.1000	6.1261
47	6.2839	6.3104	6.3369	6.3635	6.3902
48	6.5513	6.5783	6.6054	6.6325	6.6598
49	6.8242	6.8518	6.8794	6.9072	6.9349
50	7.1027	7.1309	7.1591	7.1873	7.2157
51	7.3868	7.4155	7.4443	7.4731	7.5020
52	7.6764	7.7057	7.7350	7.7644	7.7939
53	7.9717	8.0015	8.0314	8.0613	8.0913
54	8.2724	8.3028	8.3333	8.3638	8.3943
55	8.5788	8.6097	8.6407	8.6718	8.7029
56	8.8907	8.9222	8.9538	8.9854	9.0171
57	9.2082	9.2403	9.2724	9.3046	9.3368
58	9.5313	9.5639	9.5966	9.6293	9.6621
59	9.8600	9.8931	9.9263	9.9596	9.9930
60	10.1942	10.2279	10.2617	10.2955	10.3294
61	10.5339	10.5683	10.6026	10.6371	10.6715
62	10.8793	10.9142	10.9491	10.9841	11.0191
63	11.2302	11.2657	11.3011	11.3367	11.3722
64	11.5867	11.6228	11.6588	11.6948	11.7309
65	11.9488	11.9854	12.0219	12.0586	12.0953
66	12.3164	12.3536	12.3907	12.4279	12.4651
67	12.6896	12.7272	12.7649	12.8027	12.8405
68	13.0684	13.1066	13.1448	13.1831	13.2215
69	13.4527	13.4915	13.5303	13.5691	13.6080
70	13.8426	13.8819	13.9213	13.9607	14.0002
71	14.2381	14.2780	14.3179	14.3579	14.3979
72	14.6392	14.6796	14.7201	14.7606	14.8012

Table

Table II. *Areas of Circles in Wine Gallons.*

Parts of an Inch.

Diam.	.0	.1	2	.3	.4
Inch.	Areas.	Areas.	Areas.	Areas.	Areas.
10	0.3400	0.3468	0.3537	0.3607	0.3677
11	0.4114	0.4189	0.4264	0.4341	0.4418
12	0.4896	0.4978	0.5060	0.5143	0.5227
13	0.5746	0.5834	0.5924	0.6014	0.6105
14	0.6664	0.6759	0.6856	0.6952	0.7050
15	0.7650	0.7752	0.7855	0.7959	0.8063
16	0.8704	0.8814	0.8922	0.9033	0.9144
17	0.9826	0.9942	1.0058	1.0176	1.0294
18	1.1016	1.1140	1.1262	1.1385	1.1511
19	1.2279	1.2403	1.2533	1.2664	1.7796
20	1.3600	1.3736	1.3873	1.4011	1.4149
21	1.4994	1.5137	1.5280	1.5425	1.5570
22	1.6456	1.6606	1.6757	1.6907	1.7059
23	1.7986	1.8142	1.8300	1.8458	1.8617
24	1.9584	1.9747	1.9911	2.0076	2.0242
25	2.1250	2.1420	2.1591	2.1763	2.1935
26	2.2984	2.3161	2.3339	2.3517	2.3696
27	2.4786	2.4970	2.5154	2.5339	2.5525
28	2.6656	2.6846	2.7038	2.7230	2.7423
29	2.8594	2.8791	2.8989	2.9188	2.9388
30	3.0600	3.0804	3.1009	3.1215	3.1421
31	3.2674	3.2885	3.3097	3.3309	3.3522
32	3.4816	3.5034	3.5253	3.5472	3.5692
33	3.7026	3.7250	3.7476	3.7702	3.7929
34	3.9304	3.9535	3.9767	4.0000	4.0234
35	4.1650	4.1888	4.2127	4.2366	4.2607
36	4.4064	4.4309	4.4555	4.4801	4.5048
37	4.6546	4.6798	4.7050	4.7304	4.7558
38	4.9096	4.9355	4.9614	4.9874	5.0135
39	5.1714	5.1979	5.2245	5.2512	5.2780
40	5.4400	5.4672	5.4945	5.5219	5.5493
41	5.7154	5.7433	5.7713	5.7993	5.8274
42	5.9976	6.0262	6.0548	6.0836	6.1124

Parts of an Inch.

Diam.	.5	.6	.7	.8	.9
Inch.	Areas.	Areas.	Areas.	Areas.	Areas.
10	0.3748	0.3820	0.3892	0.3965	0.4039
11	0.4497	0.4575	0.4654	0.4734	0.4815
12	0.5312	0.5398	0.5483	0.5570	0.5658
13	0.6197	0.6288	0.6381	0.6475	0.6569
14	0.7148	0.7247	0.7347	0.7447	0.7548
15	0.8169	0.8274	0.8380	0.8488	0.8596
16	0.9257	0.9369	0.9482	0.9596	0.9711
17	1.0413	1.0532	1.0651	1.0773	1.0894
18	1.1637	1.1762	1.1889	1.2017	1.2145
19	1.2928	1.3061	1.3195	1.3329	1.3464
20	1.4288	1.4428	1.4569	1.4709	1.4852
21	1.5717	1.5863	1.6010	1.6158	1.6307
22	1.7213	1.7366	1.7519	1.7675	1.7830
23	1.8777	1.8936	1.9097	1.9259	1.9421
24	2.0409	2.0575	2.0743	2.0911	2.1080
25	2.2108	2.2282	2.2453	2.2631	2.2808
26	2.3876	2.4057	2.4238	2.4420	2.4603
27	2.5712	2.5900	2.6088	2.6277	2.6466
28	2.7616	2.7810	2.8005	2.8201	2.8397
29	2.9589	2.9789	2.9991	3.0193	3.0396
30	3.1628	3.1836	3.2044	3.2253	3.2463
31	3.3736	3.3951	3.4166	3.4382	3.4599
32	3.5912	3.6134	3.6356	3.6579	3.6802
33	3.8156	3.8385	3.8613	3.8843	3.9073
34	4.0468	4.0703	4.0939	4.1175	4.1412
35	4.2849	4.3090	4.3333	4.3576	4.3820
36	4.5297	4.5545	4.5794	4.6044	4.6295
37	4.7813	4.8068	4.8324	4.8580	4.8838
38	5.0397	5.0658	5.0921	5.1185	5.1449
39	5.3049	5.3317	5.3587	5.3857	5.4128
40	5.5769	5.6044	5.6320	5.6598	5.6876
41	5.8556	5.8839	5.9122	5.9406	5.9691
42	6.1412	6.1702	6.1992	6.2283	6.2574

The

The Uſe of the firſt Table of Areas of Circles above.

1. To find the Content of any Cylindrical Veſſel or Cask; ſuppoſe the Diameter 41.8 Inches, and the Length or Depth 57.5.

Againſt 41, and under .8 is the Area = 4.8662
Which multiplied by the Depth = 57.5

And the Product is the Content, Gallons of Ale, &c. —— } = 279.8

2. The Croſs Diameters and Length of an elliptical Veſſel given, to find the Content, ſuppoſing it not tapering; as admit the *Latus tranſverſum* or Axis be = 57.5, and the Diameter Conjugate 43, Length or Depth 12.3. Multiply the Croſs Diameters together, gives 2472.5, the Square Root of which is the Diameter of an equal Circle = 49.7; whoſe Area againſt 49, and under .7, is = 6.8794, which multiplied in the Length, gives the Content 84.6 Gallons Beer.

3. *The Area* of a Circle given, *to find the Diameter*; as, What is the Diameter of that Circle whoſe Area is 14? Look in the Areas for the next Number to 14, and you'll find it under .9; and towards the Left-Hand, *under Diameters* 70, ſo the Anſwer is 70.9 Inches.

4. The Content and Length of a Veſſel or Cask given, to find the Area and Diameter. There is a Cask (a Cylinder, or very near it) whoſe Content is 248.85 Gallons, and the Length 35.3; divide the Content by the Length, and the Quote is 7.05 *ferè*. This

found in the Table of Areas, the next to it is under .3, and againſt the Diameter 50 ; ſo the Anſwer is 50.3 = the Diameter, and 7.05 the Area.

5. The Uſes above, of the firſt Table of Areas of Circles in Ale-Gallons being underſtood, the Uſes of the ſecond Table are known of courſe ; for if the Diameter of a *Florence* Wine Cask, &c. (having very little or no Curvature) is = 21.3, and the Length within is 30.5, the Area in the ſecond Table, under .3, and againſt 21, is 1.5425 ; which being multiplied by the Length, the Content is = 47 Gallons of Wine.

So much for Gauging of Liquids.

CHAP. VI.

Concerning Gauging of Malt.

1. THIS is done in the Ciſtern, on the Couch, and on the Flower. *In the Ciſtern* they gauge and inch, as is taught before, and particularly as the Fruſtum of a Pyramid, under Prob. 4. of Chap. V. having reſpect to Malt-Meaſure. Thus, if the firſt Mean Dimenſions (or Length and Breadth) were 96 by 40.3, the Product is 3868.8 ; which divided by 2150.42, the Inches in a Buſhel, gives 1.8, the Area to be multiplied by ſuch a Depth as the ſaid Dimenſions, 96 and 40.3, were the Middle of, as in the ſaid Prob. 4. is particularly taught.

2. For *Gauging Malt on the Couch.* If you have a mind to be very exact, you may find ſeveral Mean Areas, as before in the laſt Rule for the Ciſtern; but they ſeldom take more than one Mean Area, and ſo proceed as in the Example following at large is ſhewn.

Inches. Inches

Admit the Length at the Top is —— } = 72, Breadth 48
At the Bottom = 60, Breadth 40.5

Areas.

The Product of the firſt two Dimenſions, gives } 3456 Inches.
Of the ſecond = 2430

Sum = 5886
½ Sum or Mean Area = 2943 Inches.

This divided by 2150.42, the ſolid Inches in a Buſhel, gives —— } = 1.3686

the Mean Area in Buſhels, which muſt be multiplied by the Number of Inches, which at any time when you come, you find the Depth of Malt to be. But,

Here you muſt obſerve, that it is not likely the Malt ſhould be of one Depth, therefore the Depth muſt be taken in ſeveral Places, as where the greateſt, the leaſt, and Mediums; the Sum of which divided by the Number of Depths taken, exhibits the Mean Depth.

As ſuppoſe I come and find the Depth of Malt in one Place =20 Inch.
in another =25
in another =22
in a fourth =21
in a fifth =23

Sum =111; a 5th of that is =22.2 the Mean Depth.

This Mean Depth multiplied by the Mean Area, will give us 30.383 Buſhels in the Couch.

3. *As for Gauging Malt on the Floor*, the Figures and Areas are ſo uncertain, that no particular Rules can be given; but tho' they be never ſo various and irregular, you will find Rules in Chap. III. to exhibit the Content: which, when you have in Inches, divide by 2150.42, to give you the Buſhels.

FINIS.

Some Clauſes in Statutes relating to the Power and Duty of Gaugers, &c. *with reſpect to Brewers and Diſtillers, Malſters, and Candles.*

GAUGERS may enter any Brew-houſe or Diſtill-houſe to gauge, as well by Night, in the Preſence of a Conſtable, as by Day. 12 *Char.* II.

Gauger to take Account of Beer, Ale, Strong Waters, *&c.* And to make Return thereof to Commiſſioners, *&c.* 12 *Char.* II.

Gauger to leave a true Copy of ſuch Return, in Writing under his Hand, with Brewer, *&c.* and ſuch Return to be a Charge on Brewer, *&c.* 12 *Char.* II.

If Brewer refuſe to permit Gauger to enter or gauge, *&c.* Gauger may forbid Brewer, *&c.* to ſell, *&c.* 12 *Char.* II.

Gauger to keep an Account and gauge the Liquors, *&c.* made by Diſtillers, and in caſe he find any other Materials mixt with Malt Drink, he ſhall charge the low Wines drawn from the Drink ſo mixt, with the Duty of 12 *d. per* Gallon. 7 & 8 *Will.* III.

Gauger may enter the Diſtilling Houſe of any Diſtiller, and take account of Materials fit for or preparing for Diſtillation; may take off the Head of any Still not at work, to ſee what

Materials it is charged with; and if at work, may ſtay in the Diſtill-Houſe till wrought off. 7 & 8 *Will.* III.

Gaugers ſhall gauge all Materials prepared for Diſtillation; if ſhall miſs any Waſh prepared from Corn, *&c.* which he found within 24 Hours before, he may charge Diſtiller with ſo much low Wines, *&c.* 7 & 8 *Will.* III.

Gauger may enter and ſtay in a Brew-houſe, to ſee a whole Guile brewed off, cleanſed and carried out. 7 & 8 *Will.* III.

Gauger may taſte Drink upon any Brewer's Dray, enter the Cellers, *&c.* of Inn keepers or Victuallers that take drink of a Brewer, and taſte the Drink.

Gauger ſhall, within three Days after every Week, leave with the Brewer, or Retailer, or their Servants, a true Copy of every Charge made by him in ſuch Week; and in Caſe of Refuſal, or if he ſhall charge any Brewer or Retailer, *&c.* more than ſuch Copy contains, forfeits ten Pounds. 7 & 8 *Will.* III. Provided that no Gauger leaving ſuch Copy of his Charge, as above directed, ſhall be liable to any of the Penaltys by former Acts impoſed for not leaving Notes of their laſt Gauges. 7 & 8 *Will.* III.

Gauger in the Day-time, and in Preſence of a Conſtable, may break open the Door or Houſe in the Poſſeſſion of any Brewer or Diſtiller, *&c.* to ſearch for private Backs, Tuns, *&c.* and break up the Ground and follow any Pipe, and break open the Door or Houſe in Poſſeſſion of any other Perſon, into which the ſame ſhall lead, making good the Houſe or

Ground ſo broke up. 7 & 8, 8 & 9, 10 & 11 *Will.* III.

If the *Gauger*, upon demand, do not leave Notes of all his Gauges, and of the Inches and Tenths, and Wants of Tuns; he ſhall forfeit, for every Neglect, the Sum of forty Shillings, with Coſt of Suit, to be recovered in the Courts of *Weſtminſter*.

Brewer uſing Sugar, *&c.* in Brewing, Penalty twenty Pounds. 12 *Anne*.

Gauger ſuſpecting any private Stills, and making Affidavit before any Juſtice, *&c.* ſetting forth the Grounds of his Suſpicion; then by Warrant from ſome Juſtice, directed to Officer of Exciſe: He, in Preſence of a Conſtable, may break open the Door where, *&c.* and ſeize the Veſſels, Spirits, *&c.* found there, and keep them in the Place where found; and if not claimed within twenty Days after Seizure, by the Owner, then forfeited, and to be ſold at the next general Day of Sale, after the Expiration of the twenty Days, a Moiety to the King, and a Moiety to the Informer; if claimed, *l.* 200 for each Still, by 10 & 11 *William* III.

Gauger ſhall keep an Account of all Melaſſes Waſh in Diſtillers hands, and upon Decreaſe thereof, charge ſo much low Wines as a 4th Part of the Waſh ſhall amount to, and charge him with ſo much proof Spirits, as $\frac{2}{5}$ of the low Wines, ſo charged, ſhall amount to. 10 & 11 *Will.* III.

Gauger may charge one fourth of low Wines from Decreaſe of Malt Waſh, and three fifths of Spirits from ſuch low Wines, by 4 Queen *Anne*.

No Officer of Exciſe to take any Money but from his Majeſty, on Forfeiture of his Office, and Incapacity for the future. 1 *Will.* & *Mary.*

Officer willingly charging low Wines, as from Corn, knowing them not to be ſuch, forfeits his Office and 10 Shillings *per* Gallon. 7 & 8 *Will.* III.

Officers may enter all Ware-Houſes, *&c.* uſed by Diſtillers, Makers or Sellers of, or Dealers in Brandy, *&c.* by 6 K. *George.*

Officers to give Certificates gratis, for Brandy ſold in entered Places. 6 K. *Geo.*

Any Perſon who obſtructs Officers, *&c.* in ſeizing Brandys, *&c.* or reſcueth or deſtroys ſuch Brandy, *&c.* ſo ſeized, forfeits 40 *l.* 8 K. *Geo.*

Secondly, As to Malt.

Gaugers, &c. upon Requeſt, in the Daytime, and in the Night with a Conſtable, may enter the Houſe, *&c.* of any Perſon making Malt, and *gauge* Ciſterns, *&c.* and other Veſſels uſed for wetting of Barley, *&c.* and take an Account of the Corn therein, and to make a Return thereof, leaving a Copy of ſuch Report under their Hands, with the Malſter, which will be a Charge on ſuch Malſter.

Gauger to meaſure Corn making into Malt, by the Gauge only, and not by the Buſhel; and ſhall leave true Copys under his Hand, under the forfeit of 40 *s.* by 12 *Anne*, thus far.

Officers may take a handful of Corn, making into Malt, to ſee if it be acreſpired; and if find it ſo, to give Notice in 48 Hours to Malſters. 6 *Geo.*

And finding Corn, ſo cloſe as it could not be unleſs forced; Malſter forfeits 2 *s.* 6 *d.* *per* Buſhel, by 12 *Anne.*

If *Malſter* refuſe to permit Gauger to enter his Houſe, or take an Account of his Malt, *&c* and to gauge his Uting Fats, or Corn therein, Penalty 20 *l.* 12 *Anne.*

If *Malſter* neglect to enter Monthly, at the Exciſe-Office, what he makes in that Month, Penalty 10 *l.* And if *Malſter* neglect to pay within three Months after Entry, forfeits double Duty, by 12 *Anne.* And if *Malſters* mix Corn of one wetting, *&c.* with other Corn of a former wetting, *&c.* forfeit 5 *s. per* Buſhel. 12 *Anne.*

Malſter ſetting up, *&c.* any Ciſtern, *&c.* without Notice in Writing at the next Exciſe-Office, forfeits 50 *l.* 12 *Anne.* And concealing Malt, forfeits 10 *s.* *per* Buſhel. 12 *Anne.*

Malſter permitting Corn to acreſpire, forfeits the Allowance of 5 *s. per* Buſhel: And Malſter wetting, *&c.* Corn on the Couch, Floor, or any other Place but his Ciſtern, *&c.* forfeits 2 *s.* 6 *d.* *per* Buſhel, by 6 *Geo.*

Thirdly, As to Candles.

Perſons making Candles, to give Notice of their Work-Houſe, at the next Office of Exciſe, and not to alter the ſame without Notice on Forfeiture of 50 *l.* 8 *Anne*. And within the Weekly-Bills, ſhall enter and clear Monthly, and in all other Places in 6 Weeks after Entry, under the Penalty of double Duty, and not to ſell till Payment, *&c.* 8. *Anne*.

Officers may enter Houſes and weigh Candles with Maker's Scales, and make Return thereof to the Commiſſioners, which Return ſhall be a Charge on the Maker, with whom a Copy ſhall be left on Demand, on Forfeiture of 40 *s.* 8. *Anne* (10 *l. per Cent.* allowed for prompt Payment, for Stock in Hand.)

Makers to find Scales for Officer to uſe, or forfeit 10 *l.*

Officer to take an Account of the Quantity of Wax, Tallow, *&c.* in Cuſtody of Maker, and afterward miſſing the ſame, is to charge the Maker, *&c.* with 108 ℔. for every 112 ℔. *&c.* ſo miſſing. 8 *Anne*.

Maker obſtructing Officers, forfeiteth 20 *l.* and by 12 *Geo.* 100 *l.*

Maker removing Candles before Account taken thereof, or Notice, *&c.* forfeits 20 *l.* 8 *Anne*.

Candles not ſurveyed, to be kept ſeparate, on Forfeiture of 5 *l.* 8 *Anne*.

Candles, or Materials for making them, concealed, forfeits 20 *l.* 8 *Anne*.

Candles found in private Melting-Houſes forfeited. 8 *Anne*.

Candles and all Materials and Utenſils liable to all Debts, Dutys in Arrear, and Forfeitures. And Candles ſpoiled in making to be defaced, and the Duty to be allowed, 8. *Ann.*

Chandler to declare to the Officer, before he begins to make a Courſe of Candles, the Number of Sticks he deſigns to make, with the Sizes of the Candles, and how many Moulds he deſigns to fill, *&c.* under the Forfeiture of 10 *l.* 12 *Anne.*

Makers of Candles (except Compounders) uſing any Melting Houſe, &c. without a particular Entry thereof, forfeits 100 *l.* 12 *Geo.*

Conſtable refuſing to go with Officer, to be preſent, where his being there, with Officer in the Execution of his Office, is by Law required, forfeits 20 *l.* 12 *Geo.*

Attempters to corrupt Officers of Exciſe, forfeit 500 *l.*

Officers to be permitted to enter, *&c.* without a Conſtable, between Morning at five, and Evening at eleven; and with a Conſtable between Evening at eleven, and Morning at 5, by 12 *Geo.*

A
SUPPLEMENT
TO THE
Gauger's Guide.

Containing Tables of the Duties.

I. THE Duty to be paid by common Brewers, without the Bills of Mortality, for the Exciſe of Strong-Beer and Ale. The Allowance of two Barrels and an half in 23, being deducted out of 5 *s.* *per* Barrel, as *per* Statutes 12 *Car.* 2; 4 *Will.* & *M*; 5 *W.* & *M*; 4 & 8 of Q. *Anne.*

II. Of the Exciſe to be paid by common Brewers, without the Weekly-Bills, for Small-Beer: The Allowance of two Barrels and an half in 23, being deducted out of the Exciſe of 1 *s.* 4 *d.* *per* Barrel, as by the ſaid Statutes above; and 1 *Will.* & *M.* for the Allowance.

III. Of the Exciſe to be paid by Victuallers for Strong-Beer and Ale at 5 *s.* *per* Barrel, as *per* ſaid Statutes.

IV. The Duty of Exciſe to be paid by Victuallers for Small-Beer at 1 *s.* 4 *d. per* Barrel.

N. B. By this Table may alſo be computed the Duty on Wax-Candles at 8 *d. per* ℔. as 8 *Anne*, taking half of any Number.

V. Table of the Duty to be paid for Malt in the Ciſtern (or Veſſel wherein it is wet or ſteeped) or within 30 Hours after it is thrown out; and the Allowance of 4 Buſhels in 20 deducted out of 6 Pence *per* Buſhel, as *per* 12 *Ann. Reg.*

VI. Of the Duty to be paid for Malt 30 Hours or more after thrown out of the Ciſtern, and before dryed; and the Allowance of 10 Buſhels in 20 deducted out of 6 *d. per* Buſhel, as *per* 12 *Ann. Reg.*

N. B. The Duty on printed Linnen of 3 *d. per* Yard Square, is alſo computed by this Table VI. *vid.* Stat. *Anne* 1712.

VII. Table of the Duty to be paid for Silver Plate at 6 *d. per* Ounce. *per* Act *An.* 1720.

N. B. By this Table is alſo computed the Duty on printed Callicoe.

VIII. Table of the Duty on Soap, at 3 Half-Pence *per* Pound. *Anno* 1712.

IX. Of the Duty on Tallow Candles of 1 *d. per* Pound, as 8 Q. *Ann.* & 9 Q. *Ann.*

X. Table of the Duty to be paid by common Brewers within the Weekly-Bills, for Strong Beer: The Allowance of three Barrels in 23, being deducted out of 5 *s. per* Barrel, as *per* 12 *Car.* 2. 1 *W.* & *M.*

XI. Of the Duty to be paid by common Brewers within the Bills of Mortality for Ale: The Allowance of two Barrels in 22 being deducted out of 5 *s. per* Barrel, as *per* Stat. under the Iſt & Xth Articles above.

XII. Of the Duty to be paid by Common Brewers within the Weekly-Bills, for Small-Beer: The Allowance of three Barrels in 23 being deducted out of 1 *s.* 4 *d. per* Barrel, as *per* the Statutes under the 1ſt & 10th Articles.

Note, Theſe Tables are more and more copious and accurate, than are Extant.

per E. HATTON, *Philomath.*

Table I. *The Duty of Exciſe to be paid by common Brewers for Strong-Beer and Ale, without the Bills of Mortality.*

Nº. of	Duty.				23	Bar.	Duty.				23
Firk.	*l.*	*s.*	*d.*	*q.*	*pts.*		*l.*	*s.*	*d.*	*q.*	*pts.*
1	—	1	1	1	11	32	7	2	7	1	5
2	—	2	2	2	22	33	7	7	—	3	3
of 3	—	3	4	0	10	34	7	11	6	1	1
Bar. 1	—	4	5	1	21	35	7	15	11	2	22
2	—	8	10	3	19	36	8	—	5	—	20
3	—	13	4	1	17	37	8	4	10	2	18
4	—	17	9	3	15	38	8	9	4	—	16
5	1	2	3	1	13	39	8	13	9	2	14
6	1	6	8	3	11	40	8	18	3	—	12
7	1	11	2	1	9	41	9	2	8	2	10
8	1	15	7	3	7	42	9	7	2	—	8
9	2	—	1	1	5	43	9	11	7	2	6
10	2	4	6	3	3	44	9	16	1	—	4
11	2	9	—	1	1	45	10	—	6	2	2
12	2	13	5	2	22	46	10	5	—	—	—
13	2	17	11	—	20	47	10	9	5	1	21
14	3	2	4	2	18	48	10	13	10	3	19
15	3	6	10	—	16	49	10	18	4	1	17
16	3	11	3	2	14	50	11	2	9	3	15
17	3	15	9	—	12	51	11	7	3	1	13
18	4	—	2	2	10	52	11	11	8	3	11
19	4	4	8	—	8	53	11	16	2	1	9
20	4	9	1	2	6	54	12	—	7	3	7
21	4	13	7	—	4	55	12	5	1	1	5
22	4	18	—	2	2	56	12	9	6	3	3
23	5	2	6	—	—	57	12	14	—	1	1
24	5	6	11	1	21	58	12	18	5	2	22
25	5	11	4	3	19	59	13	2	11	—	20
26	5	15	10	1	17	60	13	7	4	2	18
27	6	—	3	3	15	61	13	11	10	—	16
28	6	4	9	1	13	62	13	16	3	2	14
29	6	9	2	3	11	63	14	—	9	—	12
30	6	13	8	1	9	64	14	5	2	2	10
31	6	18	1	3	7	65	14	9	8	—	8

The Table for common Brewers within the Weekly-Bills, ſee the Xth Table.

The Duty of Exciſe to be paid by common Brewers without the Weekly-Bills, for Strong-Beer and Ale, continued.

Nº of	Duty.				23	Bar.	Duty.				23
Bar.	*l.*	*s.*	*d.*	*q.*	*pts.*		*l.*	*s.*	*d.*	*q.*	*pts.*
66	14	14	1	2	6	100	22	5	7	3	7
67	14	18	7	—	4	110	24	10	2	2	10
68	15	3	—	2	2	120	26	14	9	1	13
69	15	7	6	—	—	130	28	19	4	—	16
70	15	11	11	1	21	140	31	3	10	3	19
71	15	16	4	3	19	150	33	8	5	2	22
72	16	—	10	1	17	160	35	13	—	2	2
73	16	5	3	3	15	170	37	17	7	1	5
74	16	9	9	1	13	180	40	2	2	—	8
75	16	14	2	3	11	190	42	6	8	3	11
76	16	18	8	1	9	200	44	11	3	2	14
77	17	3	1	3	7	210	46	15	10	1	17
78	17	7	7	1	5	220	49	—	5	—	20
79	17	12	—	3	3	230	51	5	—	—	—
80	17	16	6	1	1	240	53	9	6	3	3
81	18	—	11	2	22	250	55	14	1	2	6
82	18	5	5	—	20	260	57	18	8	1	9
83	18	9	10	2	18	270	60	3	3	—	12
84	18	14	4	—	16	280	62	7	9	3	15
85	18	18	9	2	14	290	64	12	4	2	18
86	19	3	3	—	12	300	66	16	11	1	21
87	19	7	8	2	10	310	69	1	6	1	1
88	19	12	2	—	8	320	71	6	1	—	4
89	19	16	7	2	6	330	73	10	7	3	7
90	20	1	1	—	4	340	75	15	2	2	10
91	20	5	6	2	2	350	77	19	9	1	13
92	20	10	—	—	—	360	80	4	4	—	16
93	20	14	5	1	21	370	82	8	10	3	19
94	20	18	10	3	19	380	84	13	5	2	22
95	21	3	4	1	17	390	86	18	—	2	2
96	21	7	9	3	15	400	89	2	7	1	5
97	21	12	3	1	13	410	91	7	2	—	8
98	21	16	8	3	11	420	93	11	8	3	11
99	22	1	2	1	9	430	95	16	3	2	14

The Duty of Exciſe to be paid by common Brewers, without the Weekly-Bills. for Strong-Beer and Ale, continued.

N°of Bar.	Duty.				23	Bar.	Duty.				23
	l.	*s.*	*d.*	*q.*	*pts.*		*l.*	*s.*	*d.*	*q.*	*pts.*
440	98	—	10	1	17	780	173	16	1	—	4
450	100	5	5	—	20	790	176	—	7	3	7
460	102	10	—	—	—	800	178	5	2	2	10
470	104	14	6	3	3	810	180	9	9	1	13
480	106	19	1	2	6	820	182	14	4	—	16
490	109	3	8	1	9	830	184	18	10	3	19
500	111	8	3	—	12	840	187	3	5	2	22
510	113	12	9	3	15	850	189	8	—	2	2
520	115	17	4	2	18	860	191	12	7	1	5
530	118	1	11	1	21	870	193	17	2	—	8
540	120	6	6	1	1	880	196	1	8	3	11
550	122	11	1	—	4	890	198	6	3	2	14
560	124	15	7	3	7	900	200	10	10	1	17
570	127	—	2	2	10	910	202	15	5	0	20
580	129	4	9	1	13	920	205	—	—	—	—
590	131	9	4	—	16	930	207	4	6	3	3
600	133	13	10	3	19	940	209	9	1	2	6
610	135	18	5	2	21	950	211	13	8	1	9
620	138	3	—	2	2	960	213	18	3	—	12
630	140	7	7	1	5	970	216	2	9	3	15
640	142	12	2	—	8	980	218	7	4	2	18
650	144	16	8	3	11	990	220	11	11	1	21
660	147	1	3	2	14	1000	222	16	6	1	1
670	149	5	10	1	17	2000	445	13	—	2	2
680	151	10	5	—	20	3000	668	9	6	3	3
690	153	15	—	—	—	4000	891	6	1	—	4
700	155	19	6	3	3	5000	1114	2	7	1	5
710	158	4	1	2	6	6000	1336	19	1	2	6
720	160	8	8	1	9	7000	1559	15	7	3	7
730	162	13	3	—	12	8000	1782	12	2	—	8
740	164	17	9	3	15	9000	2005	8	8	1	9
750	167	2	4	2	18	10000	2228	5	2	2	10
760	169	6	11	1	21	20000	4456	10	5	--	20
770	171	11	6	1	1	30000	6684	15	7	3	7

N. B. The making and Uſe of this Table; See after the laſt Table.

Table II. *The Duty of Exciſe to be paid by common Brewers, for Small-Beer, without the Weekly-Bills.*

Nº. of Firk.	*l.*	*s.*	*d.*	*q.*	23 *pts.*	Bar.	*l.*	*s.*	*d.*	*q.*	23 *pts.*
1	—	—	3	2	6	32	—	18	—	1	9
2	—	—	7	—	12	33	1	19	2	2	10
3	—	—	10	2	18	34	2	—	4	3	11
Bar. 1	—	1	2	1	1	35	2	1	7	—	12
2	—	2	4	2	2	36	2	2	9	1	13
3	—	3	6	3	3	37	2	3	11	2	14
4	—	4	9	—	4	38	2	5	1	3	15
5	—	5	11	1	5	39	2	6	4	—	16
6	—	7	1	2	6	40	2	7	6	1	17
7	—	8	3	3	7	41	2	8	8	2	18
8	—	9	6	0	8	42	2	9	10	3	19
9	—	10	8	1	9	43	2	11	1	—	20
10	—	11	10	2	10	44	2	12	3	1	21
11	—	13	0	3	11	45	2	13	5	2	22
12	—	14	3	0	12	46	2	14	8	—	—
13	—	15	5	1	13	47	2	15	10	1	1
14	—	16	7	2	14	48	2	17	—	2	2
15	—	17	9	3	15	49	2	18	2	3	3
16	—	19	—	—	16	50	2	19	5	—	4
17	1	—	2	1	17	51	3	—	7	1	5
18	1	1	4	2	18	52	3	1	9	2	6
19	1	2	6	3	19	53	3	2	11	3	7
20	1	3	9	—	20	54	3	4	2	—	8
21	1	4	11	1	21	55	3	5	4	1	9
22	1	6	1	2	22	56	3	6	6	2	10
23	1	7	4	—	—	57	3	7	8	3	11
24	1	8	6	1	1	58	3	8	11	—	12
25	1	9	8	2	2	59	3	10	1	1	13
26	1	10	10	3	3	60	3	11	3	2	14
27	1	12	1	—	4	61	3	12	5	3	15
28	1	13	3	1	5	62	3	13	8	—	16
29	1	14	5	2	6	63	3	14	10	1	17
30	1	15	7	3	7	64	3	16	—	2	18
31	1	16	10	—	8	65	3	17	2	3	19

The Duty of Exciſe to be paid by common Brewers, for Small-Beer, continued.

N°of Bar.	l.	s.	d.	q.	23 pts.	Bar.	l.	s.	d.	q.	23 pts.
66	3	18	5	—	20	100	5	18	10	—	8
67	3	19	7	1	21	110	6	10	8	2	18
68	4	—	9	2	22	120	7	2	7	1	5
69	4	2	—	—	—	130	7	14	5	3	15
70	4	3	2	1	1	140	8	6	4	2	2
71	4	4	4	2	2	150	8	18	3	—	12
72	4	5	6	3	3	160	9	10	1	2	22
73	4	6	9	—	4	170	10	2	—	1	9
74	4	7	11	1	5	180	10	13	10	3	19
75	4	9	1	2	6	190	11	5	9	2	6
76	4	10	3	3	7	200	11	17	8	—	16
77	4	11	6	—	8	210	12	9	6	3	3
78	4	12	8	1	9	220	13	1	5	1	13
79	4	13	10	2	10	230	13	13	4	—	—
80	4	15	—	3	11	240	14	5	2	2	10
81	4	16	3	—	12	250	14	17	1	0	20
82	4	17	5	1	13	260	15	8	11	3	7
83	4	18	7	2	14	270	16	—	10	1	17
84	4	19	9	3	15	280	16	12	9	0	4
85	5	1	—	—	16	290	17	4	7	2	14
86	5	2	2	1	17	300	17	16	6	1	1
87	5	3	4	2	18	310	18	8	4	3	11
88	5	4	6	3	19	320	19	0	3	1	21
89	5	5	9	—	20	330	19	12	2	—	8
90	5	6	11	1	21	340	20	4	—	2	18
91	5	8	1	2	22	350	20	15	11	1	5
92	5	9	4	—	—	360	21	7	9	3	15
93	5	10	6	1	1	370	21	19	8	2	2
94	5	11	8	2	2	380	22	11	7	—	12
95	5	12	10	3	3	390	23	3	5	2	22
96	5	14	1	—	4	400	23	15	4	1	9
97	5	15	3	1	5	410	24	7	2	3	19
98	5	16	5	2	6	420	24	19	1	2	6
99	5	17	7	3	7	430	25	11	—	—	16

The

The Duty of Exciſe to be paid by common Brewers, for Small-Beer, continued.

N°of Bar.	*l.*	*s.*	*d.*	*q.*	23 *pts.*
440	26	2	10	3	3
450	26	14	9	1	13
460	27	6	8	—	—
470	27	18	6	2	10
480	28	10	5	—	20
490	29	2	3	3	7
500	29	14	2	1	17
510	30	6	1	—	4
520	30	17	11	2	14
530	31	9	10	1	1
540	32	1	8	3	11
550	32	13	7	1	21
560	33	5	6	—	8
570	33	17	4	2	18
580	34	9	3	1	5
590	35	1	1	3	15
600	35	13	—	2	2
610	36	4	11	—	12
620	36	16	9	2	22
630	37	8	8	1	9
640	38	—	6	3	19
650	38	12	5	2	6
660	39	4	4	—	16
670	39	16	2	3	3
680	40	8	1	1	13
690	41	—	—	—	—
700	41	11	10	2	10
710	42	3	9	—	20
720	42	15	7	3	7
730	43	7	6	1	17
740	43	19	5	—	4
750	44	11	3	2	14
760	45	3	2	1	1
770	45	15	—	3	11
Bar.	*l*	*s.*	*d.*	*q.*	23 *pts.*
780	46	6	11	1	21
790	46	18	10	—	8
800	47	10	8	2	18
810	48	2	7	1	5
820	48	14	5	3	15
830	49	6	4	2	2
840	49	18	3	—	12
850	50	10	1	2	22
860	51	2	—	1	9
870	51	13	10	3	19
880	52	5	9	2	6
890	52	17	8	—	16
900	53	9	6	3	3
910	54	1	5	1	13
920	54	13	4	—	—
930	55	5	2	2	10
940	55	17	1	—	20
950	56	8	11	3	7
960	57	—	10	1	17
970	57	12	9	—	4
980	58	4	7	2	14
990	58	16	6	1	1
1000	59	8	4	3	11
2000	118	16	9	2	22
3000	178	5	2	2	10
4000	237	13	7	1	21
5000	297	2	—	1	9
6000	356	10	5	—	20
7000	415	18	10	—	8
8000	475	7	2	3	19
9000	534	15	7	3	7
10000	594	4	—	2	18
20000	1188	8	1	1	13
30000	1782	12	2	—	8

Table III. *The Duty of Exciſe to be paid by Victuallers for Strong-Beer and Ale.*

Nº of Bar.	*l.*	*s.*	Bar.	*l.*	*s.*	Bar.	*l.*	*s.*	Bar.	*l.*	*s.*
1	0	5	34	8	10	67	16	15	100	25	—
2	0	10	35	8	15	68	17	—	110	27	10
3	0	15	36	9	—	69	17	5	120	30	—
4	1	—	37	9	5	70	17	10	130	32	10
5	1	5	38	9	10	71	17	15	140	35	—
6	1	10	39	9	15	72	18	—	150	37	10
7	1	15	40	10	—	73	18	5	160	40	—
8	2	—	41	10	5	74	18	10	170	42	10
9	2	5	42	10	10	75	18	15	180	45	—
10	2	10	43	10	15	76	19	—	190	47	10
11	2	51	44	11	—	77	19	5	200	50	—
12	3	—	45	11	5	78	19	10	210	52	10
13	3	5	46	11	10	79	19	15	220	55	—
14	3	10	47	11	15	80	20	—	230	57	10
15	3	15	48	12	—	81	20	5	240	60	—
16	4	—	49	12	5	82	20	10	250	62	10
17	4	5	50	12	10	83	20	15	300	75	—
18	4	10	51	12	15	84	21	—	350	87	10
19	4	15	52	13	—	85	21	5	400	100	—
20	5	—	53	13	5	86	21	10	450	112	10
21	5	5	54	13	10	87	21	15	500	125	—
22	5	10	55	13	15	88	22	—	550	137	10
23	5	15	56	14	—	89	22	5	600	150	—
24	6	—	57	14	5	90	22	10	650	162	10
25	6	5	58	14	10	91	22	15	700	175	—
26	6	10	59	14	15	92	23	—	750	187	10
27	6	15	60	15	—	93	23	5	800	200	—
28	7	—	61	15	5	94	23	10	850	212	10
29	7	5	62	15	10	95	23	15	900	225	—
30	7	10	63	15	15	96	24	—	950	237	10
31	7	15	64	16	—	97	24	5	1000	250	—
32	8	—	65	16	5	98	24	10	2000	500	—
33	8	5	66	16	10	99	24	17	3000	750	—

Note, That a Firkin (or 4th Part of a Barrel) is of conſequence, 15 *d.* 2 Firk. 2 *s.* 6 *d.* and 3 Firk. 3 *s.* 9 *d.*

Table IV. *The Duty of Exciſe to be paid by Victuallers for Small-Beer at* 1 s. 4 d. per *Barrel; and for Wax Candles at* 8 d. (*taking* ½.)

Nº of Bar.	*l.*	*s.*	*d.*	Bar.	*l.*	*s.*	*d.*
1	—	1	4	34	2	5	4
2	—	2	8	35	2	6	8
3	—	4	—	36	2	8	—
4	—	5	4	37	2	9	4
5	—	6	8	38	2	10	8
6	—	8	—	39	2	12	—
7	—	9	4	40	2	13	4
8	—	10	8	41	2	14	8
9	—	12	—	42	2	16	—
10	—	13	4	43	2	17	4
11	—	14	8	44	2	18	8
12	—	16	—	45	3	—	—
13	—	17	4	46	3	1	4
14	—	18	8	47	3	2	8
15	1	—	—	48	3	4	—
16	1	1	4	49	3	5	4
17	1	2	8	50	3	6	8
18	1	4	—	51	3	8	—
19	1	5	4	52	3	9	4
20	1	6	8	53	3	10	8
21	1	8	—	54	3	12	—
22	1	9	4	55	3	13	4
23	1	10	8	56	3	14	8
24	1	12	—	57	3	16	—
25	1	13	4	58	3	17	4
26	1	14	8	59	3	18	8
27	1	16	—	60	4	—	—
28	1	17	4	61	4	1	4
29	1	18	8	62	4	2	8
30	2	—	—	63	4	4	—
31	2	1	4	64	4	5	4
32	2	2	8	65	4	6	8
33	2	4	—	66	4	8	—

The Duty of Exciſe to be paid by Victuallers for Small-Beer, at 1 s. 4d. per *Barrel; and for Wax Candles at* 8d. (*taking* $\frac{1}{2}$) *continued.*

Nº of Bar.	*l.*	*s.*	*d.*	Bar.	*l.*	*s.*	*d.*
67	4	9	4	100	6	13	4
68	4	10	8	110	7	6	8
69	4	12	—	120	8	—	—
70	4	13	4	130	8	13	4
71	4	14	8	140	9	6	8
72	4	16	—	150	10	—	—
73	4	17	4	160	10	13	4
74	4	18	8	170	11	6	8
75	5	—	—	180	12	—	—
76	5	1	4	190	12	13	4
77	5	2	8	200	13	6	8
78	5	4	—	210	14	—	—
79	5	5	4	220	14	13	4
80	5	6	8	230	15	6	8
81	5	8	—	240	16	—	—
82	5	9	4	250	16	13	4
83	5	10	8	300	20	—	—
84	5	12	—	350	23	13	8
85	5	13	4	400	26	6	4
86	5	14	8	450	30	—	—
87	5	16	—	500	33	6	8
88	5	17	4	550	36	13	4
89	5	18	8	600	40	—	—
90	6	—	—	650	43	6	8
91	6	1	4	700	46	13	4
92	6	2	8	750	50	—	—
93	6	4	—	800	53	6	8
94	6	5	4	850	56	13	4
95	6	6	8	900	60	—	—
96	6	8	—	950	63	6	8
97	6	9	4	1000	66	13	4
98	6	10	8	2000	133	6	8
99	6	12	—	3000	200	—	—

Note, 1 Firkin is 4 *d.* 2 is 8 *d.* and 3 is 1 *s.*

Table V. *The Duty payable for Malt in the Ciſtern, or within* 30 *Hours after it is thrown out of it.*

Nº. of Buſh.	*s.*	*d.*	*q.*	5 *pts.*	Buſh.	*l.*	*s.*	*d.*	*q.*	5 *pts.*
1	—	4	3	1	34	—	13	7	—	4
2	—	9	2	2	35	—	14	—	—	—
3	1	2	1	3	36	—	14	4	3	1
4	1	7	—	4	37	—	14	9	2	2
5	2	—	—	—	38	—	15	2	1	3
6	2	4	3	1	39	—	15	7	—	1
7	2	9	2	2	40	—	16	—	—	—
8	3	2	1	3	41	—	16	4	3	1
9	3	7	0	4	42	—	16	9	2	2
10	4	—	—	—	43	—	17	2	1	3
11	4	4	3	1	44	—	17	7	—	4
12	4	9	2	2	45	—	18	—	—	—
13	5	2	1	3	46	—	18	4	3	1
14	5	7	—	4	47	—	18	9	2	2
15	6	—	—	—	48	—	19	2	1	3
16	6	4	3	1	49	—	19	7	—	4
17	6	9	2	2	50	1	—	—	—	—
18	7	2	1	3	51	1	—	4	3	1
19	7	7	—	4	52	1	—	9	2	2
20	8	—	—	—	53	1	1	2	1	3
21	8	4	3	1	54	1	1	7	—	4
22	8	9	2	2	55	1	2	—	—	—
23	9	2	1	3	56	1	2	4	3	1
24	9	7	—	4	57	1	2	9	2	2
25	10	—	—	—	58	1	3	2	1	3
26	10	4	3	1	59	1	3	7	—	4
27	10	9	2	2	60	1	4	—	—	—
28	11	2	1	3	61	1	4	4	3	1
29	11	7	—	4	62	1	4	9	2	2
30	12	—	—	—	63	1	5	2	1	3
31	12	4	3	1	64	1	5	7	—	4
32	—	12	9	22	65	1	6	—	—	—
33	—	13	2	13	66	1	6	4	3	1

The Duty payable for Malt in the Ciſtern, or within 30 Hours after it is thrown out of it.

Nº of Buſh.	*l.*	*s.*	*d.*	*q.*	5 *pts*	Buſh.	*l.*	*s.*
67	1	6	9	2	2	100	2	—
68	1	7	2	1	3	110	2	4
69	1	7	7	—	4	120	2	8
70	1	8	—	—	—	130	2	12
71	1	8	4	3	1	140	2	16
72	1	8	9	2	2	150	3	—
73	1	9	2	1	3	160	3	4
74	1	9	7	—	4	170	3	8
75	1	10	—	—	—	180	3	12
76	1	10	4	3	1	190	3	16
77	1	10	9	2	2	200	4	—
78	1	11	2	1	3	210	4	4
79	1	11	7	—	4	220	4	8
80	1	12	—	—	—	230	4	12
81	1	12	4	3	1	240	4	16
82	1	12	9	2	2	250	5	—
83	1	13	2	1	3	300	6	—
84	1	13	7	—	4	400	8	—
85	1	14	—	—	—	500	10	—
86	1	14	4	3	1	600	12	—
87	1	14	9	2	2	700	14	—
88	1	15	2	1	3	800	16	—
89	1	15	7	—	4	900	18	—
90	1	16	—	—	—	1000	20	—
91	1	16	4	3	1	2000	40	—
92	1	16	9	2	2	3000	60	—
93	1	17	2	1	3	4000	80	—
94	1	17	7	—	4	5000	100	—
95	1	18	—	—	—	6000	120	—
96	1	18	4	3	1	7000	140	—
97	1	18	9	2	2	8000	160	—
98	1	19	2	1	3	9000	180	—
99	1	19	7	—	4	10000	200	—

Table VI. *The Duty to be paid for Malt* 30 *Hours or more after it is thrown out of the Ciſtern*; *or for printed Linnen at* 3 d. per *Yard.*

Nº of Buſh.	*s.*	*d.*	Buſh.	*s.*	*d.*
1	—	3	34	8	6
2	—	6	35	8	9
3	—	9	36	9	—
4	1	—	37	9	3
5	1	3	38	9	6
6	1	6	39	9	9
7	1	9	40	10	—
8	2	—	41	10	3
9	2	3	42	10	6
10	2	6	43	10	9
11	2	9	44	11	—
12	3	—	45	11	3
13	3	3	46	11	6
14	3	6	47	11	9
15	3	9	48	12	—
16	4	—	49	12	3
17	4	3	50	12	6
18	4	6	51	12	9
19	4	9	52	13	—
20	5	—	53	13	3
21	5	3	54	13	6
22	5	6	55	13	9
23	5	9	56	14	—
24	6	—	57	14	3
25	6	3	58	14	6
26	6	6	59	14	9
27	6	9	60	15	—
28	7	—	61	15	3
29	7	3	62	15	6
30	7	6	63	15	9
31	7	9	64	16	—
32	8	—	65	16	3
33	8	3	66	16	6

The Duty to be paid for Malt 30 Hours or more after it is thrown out of the Ciſtern, or for printed Linnen at 3 d. per *Yard.*

Nº of Buſh.	*l.*	*s.*	*d.*	Buſh.	*l.*	*s.*	*d.*
67	—	16	9	100	1	5	—
68	—	17	—	110	1	7	6
69	—	17	3	120	1	10	—
70	—	17	6	130	1	12	6
71	—	17	9	140	1	15	—
72	—	18	—	150	1	17	6
73	—	18	3	160	2	—	—
74	—	18	6	170	2	2	6
75	—	18	9	180	2	5	—
76	—	19	—	190	2	7	6
77	—	19	3	200	2	10	—
78	—	19	6	210	2	12	6
79	—	19	9	220	2	15	—
80	1	—	—	230	2	17	6
81	1	—	3	240	3	—	—
82	1	—	6	250	3	2	6
83	1	—	9	300	3	15	—
84	1	1	—	400	5	—	—
85	1	1	3	500	6	5	—
86	1	1	6	600	7	10	—
87	1	1	9	700	8	15	—
88	1	2	—	800	10	—	—
89	1	2	3	900	11	5	—
90	1	2	6	1000	12	10	—
91	1	2	9	2000	25	—	—
92	1	3	—	3000	37	10	—
93	1	3	3	4000	50	—	—
94	1	3	6	5000	62	10	—
95	1	3	9	6000	75	—	—
96	1	4	—	7000	87	10	—
97	1	4	3	8000	100	—	—
98	1	4	6	9000	112	10	—
99	1	4	9	10000	125	—	—

Table VII. *The Duty to be paid for Plate, at* 6d. *per Ounce, or for printed Callico at* 6d. *per Yard.*

N° of Ounc.	*l.*	*s.*	*d.*	Oun.	*l.*	*s.*	*d.*
25	—	12	6	58	1	9	—
26	—	13	—	59	1	9	6
27	—	13	6	60	1	10	—
28	—	14	—	61	1	10	6
29	—	14	6	62	1	11	—
30	—	15	—	63	1	11	6
31	—	15	6	64	1	12	—
32	—	16	—	65	1	12	6
33	—	16	6	66	1	13	—
34	—	17	—	67	1	13	6
35	—	17	6	68	1	14	—
36	—	18	—	69	1	14	6
37	—	18	6	70	1	15	—
38	—	19	—	71	1	15	6
39	—	19	6	72	1	16	—
40	1	—	—	73	1	16	6
41	1	—	6	74	1	17	—
42	1	1	—	75	1	17	6
43	1	1	6	76	1	18	—
44	1	2	—	77	1	18	6
45	1	2	6	78	1	19	—
46	1	3	—	79	1	19	6
47	1	3	6	80	2	—	—
48	1	4	—	81	2	—	6
49	1	4	6	82	2	1	—
50	1	5	—	83	2	1	6
51	1	5	6	84	2	2	—
52	1	6	—	85	2	2	6
53	1	6	6	86	2	3	—
54	1	7	—	87	2	3	6
55	1	7	6	88	2	4	—
56	1	8	—	89	2	4	6
57	1	8	6	90	2	5	—

The Duty to be paid for Plate at 6 d. *per Ounce, or for printed Callico at* 6 d. *per Yard.*

Nº of Ounc.	*l.*	*s.*	*d.*	Oun.	*l.*	*s.*	*d.*
91	2	5	6	340	8	10	—
92	2	6	—	350	8	15	—
93	2	6	6	360	9	—	—
94	2	7	—	370	9	5	—
95	2	7	6	380	9	10	—
96	2	8	—	390	9	15	—
97	2	8	6	400	10	—	—
98	2	9	—	410	10	5	—
99	2	9	6	420	10	10	—
100	2	10	—	430	10	15	—
110	2	15	—	440	11	—	—
120	3	—	—	450	11	5	—
130	3	5	—	460	11	10	—
140	3	10	—	470	11	15	—
150	3	15	—	480	12	—	—
160	4	—	—	490	12	5	—
170	4	5	—	500	12	10	—
180	4	10	—	600	15	—	—
190	4	15	—	700	17	10	—
200	5	—	—	800	20	—	—
210	5	5	—	900	22	10	—
220	5	10	—	1000	25	—	—
230	5	15	—	2000	50	—	—
240	6	—	—	3000	75	—	—
250	6	5	—	4000	100	—	—
260	6	10	—	5000	125	—	—
270	6	15	—	6000	150	—	—
280	7	—	—	7000	175	—	—
290	7	5	—	8000	200	—	—
300	7	10	—	9000	225	—	—
310	7	15	—	10000	250	—	—
320	8	—	—	20000	500	—	—
330	8	5	—	30000	750	—	—

Table VIII. *The Duty on Soap at three Half-Pence* per ℔.

Poun. of Soap.	*s.*	*d.*	*q.*	Pound. of Soap.	*s.*	*d.*	*q.*
25	3	1	2	58	7	3	—
26	3	3	—	59	7	4	2
27	3	4	2	60	7	6	—
28	3	6	—	61	7	7	2
29	3	7	2	62	7	9	—
30	3	9	—	63	7	10	2
31	3	10	2	64	8	—	—
32	4	—	—	65	8	1	2
33	4	1	2	66	8	3	—
34	4	3	—	67	8	4	2
35	4	4	2	68	8	6	—
36	4	6	—	69	8	7	2
37	4	7	2	70	8	9	—
38	4	9	—	71	8	10	2
39	4	10	2	72	9	—	—
40	5	—	—	73	9	1	2
41	5	1	2	74	9	3	—
42	5	3	—	75	9	4	2
43	5	4	2	76	9	6	—
44	5	6	—	77	9	7	2
45	5	7	2	78	9	9	—
46	5	9	—	79	9	10	2
47	5	10	2	80	10	—	—
48	6	—	—	81	10	1	2
49	6	1	2	82	10	3	—
50	6	3	—	83	10	4	2
51	6	4	2	84	10	6	—
52	6	6	—	85	10	7	2
53	6	7	2	86	10	9	—
54	6	9	—	87	10	10	2
55	6	10	2	88	11	—	—
56	7	—	—	89	11	1	2
57	7	1	2	90	11	3	—

The Duty on Soap at three Half-Pence per ℔.

Poun. of Soap.	*l.*	*s.*	*d.*	*q.*	Pound. of Soap.	*l.*	*s.*	*d.*
91	—	11	4	2	340	2	2	6
92	—	11	6	—	350	2	3	9
93	—	11	7	2	360	2	5	—
94	—	11	9	—	370	2	6	3
95	—	11	10	2	380	2	7	6
96	—	12	—	—	390	2	8	9
97	—	12	1	2	400	2	10	—
98	—	12	3	—	410	2	11	3
99	—	12	4	2	420	2	12	6
100	—	12	6	—	430	2	13	9
110	—	13	9	—	440	2	15	—
120	—	15	—	—	450	2	16	3
130	—	16	3	—	460	2	17	6
140	—	17	6	—	470	2	18	9
150	—	18	9	—	480	3	—	—
160	1	—	—	—	490	3	1	3
170	1	1	3	—	500	3	2	6
180	1	2	6	—	600	3	15	—
190	1	3	9	—	700	4	7	6
200	1	5	—	—	800	5	—	—
210	1	6	3	—	900	5	12	6
220	1	7	6	—	1000	6	5	—
230	1	8	9	—	2000	12	10	—
240	1	10	—	—	3000	18	15	—
250	1	11	3	—	4000	25	—	—
260	1	12	6	—	5000	31	5	—
270	1	13	9	—	6000	37	10	—
280	1	15	—	—	7000	43	15	—
290	1	16	3	—	8000	50	—	—
300	1	17	6	—	9000	56	5	—
310	1	18	9	—	10000	62	10	—
320	2	—	—	—	20000	125	—	—
330	2	1	3	—	30000	187	10	—

Table IX. *The Duty to be paid for Tallow-Candles at* 1 d. per *Pound.*

Pounds of Can.	*s.*	*d.*	℔ of Can.	*l.*	*s.*	*d.*
61	5	1	94	—	7	10
62	5	2	95	—	7	11
63	5	3	96	—	8	—
64	5	4	97	—	8	1
65	5	5	98	—	8	2
66	5	6	99	—	8	3
67	5	7	100	—	8	4
68	5	8	110	—	9	2
69	5	9	120	—	10	—
70	5	10	130	—	10	10
71	5	11	140	—	11	8
72	6	—	150	—	12	6
73	6	1	160	—	13	4
74	6	2	170	—	14	2
75	6	3	180	—	15	—
76	6	4	190	—	15	10
77	6	5	200	—	16	8
78	6	6	210	—	17	6
79	6	7	220	—	18	4
80	6	8	230	—	19	2
81	6	9	240	1	—	—
82	6	10	250	1	—	10
83	6	11	260	1	1	8
84	7	—	270	1	2	6
85	7	1	280	1	3	4
86	7	2	290	1	4	2
87	7	3	300	1	5	—
88	7	4	310	1	5	10
89	7	5	320	1	6	8
90	7	6	330	1	7	6
91	7	7	340	1	8	4
92	7	8	350	1	9	2
93	7	9	360	1	10	—

The Duty to be paid for Tallow-Candles at 1 d. *per Pound.*

Pounds of Can.	*l.*	*s.*	*d.*	℔. of Can.	*l.*	*s.*	*d.*
370	1	10	10	700	2	18	4
380	1	11	8	710	2	19	2
390	1	12	6	720	3	—	—
400	1	13	4	730	3	—	10
410	1	14	2	740	3	1	8
420	1	15	—	750	3	2	6
430	1	15	10	760	3	3	4
440	1	16	8	770	3	4	2
450	1	17	6	780	3	5	—
460	1	18	4	790	3	5	10
470	1	19	2	800	3	6	8
480	2	—	—	810	3	7	6
490	2	—	10	820	3	8	4
500	2	1	8	830	3	9	2
510	2	2	6	840	3	10	—
520	2	3	4	850	3	10	10
530	2	4	2	860	3	11	8
540	2	5	—	870	3	12	6
550	2	5	10	880	3	13	4
560	2	6	8	890	3	14	2
570	2	7	6	900	3	15	—
580	2	8	4	910	3	15	10
590	2	9	2	920	3	16	8
600	2	10	—	930	3	17	6
610	2	10	10	940	3	18	4
620	2	11	8	950	3	19	2
630	2	12	6	960	4	—	—
640	2	13	4	970	4	—	10
650	2	14	2	980	4	1	8
660	2	15	—	990	4	2	6
670	2	15	10	1000	4	3	4
680	2	16	8	2000	8	6	8
690	2	17	6	3000	12	10	—

N.B. 2000 doubled is 4000 or *l.* 16 : 13 : 4, } and 6000
or 5000, take 3000 and 2000 = 20 : 16 : 8 } is double
(3000, or *l.* 25, *&c.*

And for Wax-Candles: See the Titles of the Tables in this Supplement and the Uſe following.

Table X. *The Duty of Exciſe to be paid by common Brewers within the Bills of Mortality, for Strong-Beer.*

N°of Bar.	*l.*	*s.*	*d.*	*q.*	23 *pts.*	Bar.	*l.*	*s.*	*d.*	*q.*	23 *pts.*
1	—	4	4	—	16	34	7	7	9	3	15
2	—	8	8	1	9	35	7	12	2	—	8
3	—	13	—	2	2	36	7	16	6	1	1
4	—	17	4	2	18	37	8	—	10	1	17
5	1	1	8	3	11	38	8	5	2	2	10
6	1	6	1	—	4	39	8	9	6	3	3
7	1	10	5	—	20	40	8	13	10	3	19
8	1	14	9	1	13	41	8	18	3	—	12
9	1	19	1	2	6	42	9	2	7	1	5
10	2	3	5	2	22	43	9	6	11	1	21
11	2	7	9	3	15	44	9	11	3	2	14
12	2	12	2	—	8	45	9	15	7	3	7
13	2	16	6	1	1	46	10	—	—	—	—
14	3	—	10	1	17	47	10	4	4	—	16
15	3	5	2	2	10	48	10	8	8	1	9
16	3	9	6	3	3	49	10	13	—	2	2
17	3	13	10	3	19	50	10	17	4	2	18
18	3	18	3	—	12	51	11	1	8	3	11
19	4	2	7	1	5	52	11	6	1	—	4
20	4	6	11	1	21	53	11	10	5	—	20
21	4	11	3	2	14	54	11	14	9	1	13
22	4	15	7	3	7	55	11	19	1	2	6
23	5	—	—	—	—	56	12	3	5	2	22
24	5	4	4	—	16	57	12	7	9	3	15
25	5	8	8	1	9	58	12	12	2	—	8
26	5	13	—	2	2	59	12	16	6	1	1
27	5	17	4	2	18	60	13	—	10	1	17
28	6	1	8	3	11	61	13	5	2	2	10
29	6	6	1	—	4	62	13	9	6	3	3
30	6	10	5	—	20	63	13	13	10	3	19
31	6	14	9	1	13	64	13	18	3	—	12
32	6	19	1	2	6	65	14	2	7	1	5
33	7	3	5	2	22	66	14	6	11	1	21

The Duties of Exciſe to be paid by common Brewers within the Bills of Mortality, for Strong-Beer, continued.

N°of Bar.	*l.*	*s.*	*d.*	*q.*	23 *pts.*	Bar.	*l.*	*s.*	*d.*	*q.*	23 *pts.*
67	14	11	3	2	14	100	21	14	9	1	13
68	14	15	7	3	7	110	23	18	3	—	12
69	15	—	—	—	—	120	26	1	8	3	11
70	15	4	4	—	16	130	28	5	2	2	10
71	15	8	8	1	9	140	30	8	8	1	9
72	15	13	—	2	2	150	32	12	2	—	8
73	15	17	4	2	18	160	34	15	7	3	7
74	16	1	8	3	11	170	36	19	1	2	6
75	16	6	1	—	4	180	39	2	7	1	5
76	16	10	5	—	20	190	41	6	1	—	4
77	16	14	9	1	13	200	43	9	6	3	3
78	16	19	1	2	6	210	45	13	—	2	2
79	17	3	5	2	22	220	47	16	6	1	1
80	17	7	9	3	15	230	50	—	—	—	—
81	17	12	2	—	8	240	52	3	5	2	22
82	17	16	6	1	1	250	54	6	11	1	21
83	18	—	10	1	17	260	56	10	5	—	20
84	18	5	2	2	10	270	58	13	10	3	19
85	18	9	6	3	3	280	60	17	4	2	18
86	18	13	10	3	19	290	63	—	10	1	17
87	18	18	3	—	12	300	65	4	4	—	16
88	19	2	7	1	5	310	67	7	9	3	15
89	19	6	11	1	21	320	69	11	3	2	14
90	19	11	3	2	14	330	71	14	9	1	13
91	19	15	7	3	7	340	73	18	3	—	12
92	20	—	—	—	—	350	76	1	8	3	11
93	20	4	4	—	16	360	78	5	2	2	10
94	20	8	8	1	9	370	80	8	8	1	9
95	20	13	—	2	2	380	82	12	2	—	8
96	20	17	4	2	18	390	84	15	7	3	7
97	21	1	8	3	11	400	86	19	1	2	6
98	21	6	1	—	4	410	89	2	7	1	5
99	21	10	5	—	20	420	91	6	1	—	4

The Duty of Exciſe to be paid by common Brewers within the Bills of Mortality, for Strong-Beer, continued.

N°of Bar.	*l.*	*s.*	*d.*	*q.*	*pts.*	Bar.	*l.*	*s.*	*d.*	*q.*	23 *pts.*
430	93	9	6	3	3	760	165	4	4	—	16
440	95	13	—	2	2	770	167	7	9	3	15
450	97	16	6	1	1	780	169	11	3	2	14
460	100	—	—	—	—	790	171	14	9	1	13
470	102	3	5	2	22	800	173	18	3	—	12
480	104	6	11	1	21	810	176	1	8	3	11
490	106	10	5	—	20	820	178	5	2	2	10
500	108	13	10	3	19	830	180	8	8	1	9
510	110	17	4	2	18	840	182	12	2	—	8
520	113	—	10	1	17	850	184	15	7	3	7
530	115	4	4	—	16	860	186	19	1	2	6
540	117	7	9	3	15	870	189	2	7	1	5
550	119	11	3	2	14	880	191	6	1	—	4
560	121	14	9	1	13	890	193	9	6	3	3
570	123	18	3	—	12	900	195	13	—	2	2
580	126	1	8	3	11	910	197	16	6	1	1
590	128	5	2	2	10	920	200	—	—	—	—
600	130	8	8	1	9	930	202	3	5	2	22
610	132	12	2	—	8	940	204	6	11	1	21
620	134	15	7	3	7	950	206	10	5	—	20
630	136	19	1	2	6	960	208	13	10	3	19
640	139	2	7	1	5	1000	217	7	9	3	15
650	141	6	1	—	4	2000	434	15	7	3	7
660	143	9	6	3	3	3000	652	3	5	2	22
670	145	13	—	2	2	4000	869	11	3	2	14
680	147	16	6	1	1	5000	1086	19	1	2	6
690	150	—	—	—	—	6000	1304	6	11	1	21
700	152	3	5	2	22	7000	1521	14	9	1	13
710	154	6	11	1	21	8000	1739	2	7	1	5
720	156	10	5	—	20	9000	1956	10	5	—	20
730	158	13	10	3	19	10000	2173	18	3	—	12
740	160	17	4	2	18	20000	4347	16	6	1	1
750	163	—	10	1	17	30000	6521	14	9	1	13

	s.	*d.*	*q.*		
Note 1 Firkin is	1	1	0	4	Duty to be paid.
2 —— is	2	2	0	8	
3 —— is	3	3	0	12	

Table XI. *The Duty of Exciſe to be paid by common Brewers within the Bills of Mortality, for Ale.*

N° of					23						23
Bar.	*l.*	*s.*	*d.*	*q.*	*pts.*	Bar.	*l.*	*s.*	*d.*	*q.*	*pts.*
1	—	4	6	2	4	35	7	19	1	—	8
2	—	9	1	—	8	36	8	3	7	2	12
3	—	13	7	2	12	37	8	8	2	—	16
4	—	18	2	—	16	38	8	12	8	2	20
5	1	2	8	2	20	39	8	17	3	1	2
6	1	7	3	1	2	40	9	1	9	3	6
7	1	11	9	3	6	41	9	6	4	1	10
8	1	16	4	1	10	42	9	10	10	3	14
9	2	—	10	3	14	43	9	15	5	1	18
10	2	5	5	1	18	44	10	—	—	—	—
11	2	10	—	—	—	45	10	4	6	2	4
12	2	14	6	2	4	46	10	9	1	—	8
13	2	19	1	—	8	47	10	13	7	2	12
14	3	3	7	2	12	48	10	18	2	—	16
15	3	8	2	—	16	49	11	2	8	2	20
16	3	12	8	2	20	50	11	7	3	1	2
17	3	17	3	1	2	51	11	11	9	3	6
18	4	1	9	3	6	52	11	16	4	1	10
19	4	6	4	1	10	53	12	—	10	3	14
20	4	10	10	3	14	54	12	5	5	1	18
21	4	15	5	1	18	55	12	10	—	—	—
22	5	—	—	—	—	56	12	14	6	2	4
23	5	4	6	2	4	57	12	19	1	—	8
24	5	9	1	—	8	58	13	3	7	2	12
25	5	13	7	2	12	59	13	8	2	—	16
26	5	18	2	—	16	60	13	12	8	2	20
27	6	2	8	2	20	61	13	17	3	1	2
28	6	7	3	1	2	62	14	1	9	3	6
29	6	11	9	3	6	63	14	6	4	1	10
30	6	16	4	1	10	64	14	10	10	3	14
31	7	—	10	3	14	65	14	15	5	1	18
32	7	5	5	1	18	66	15	—	—	—	—
33	7	10	—	—	—	67	15	4	6	2	4
34	7	14	6	2	4	68	15	9	1	—	8

The

The Duty of Exciſe to be paid by common Brewers within the Bills of Mortality, for Ale, continued.

N° of Bar.	*l.*	*s.*	*d.*	*q.*	23 *pts.*	Bar.	*l.*	*s.*	*d.*	*q.*	23 *pts.*
69	15	13	7	2	12	500	113	12	8	2	20
70	15	18	2	—	16	600	136	7	3	1	2
71	16	2	8	2	20	700	159	1	9	3	6
72	16	7	3	1	2	800	181	16	4	1	10
73	16	11	9	3	6	900	204	10	10	3	14
74	16	16	4	1	10	1000	227	5	5	1	18
75	17	—	10	3	14	2000	454	10	10	3	14
76	17	5	5	1	18	3000	681	16	4	1	10
77	17	10	—	—	—	4000	909	1	9	3	6
78	17	14	6	2	4	5000	1136	7	3	1	2
79	17	19	1	—	8	6000	1363	12	8	2	20
80	18	3	7	2	12	7000	1590	18	2	—	16
90	20	9	1	—	8	8000	1818	3	7	2	12
100	22	14	6	2	4	9000	2045	9	1	—	8
200	45	9	1	—	8	10000	2272	14	6	2	4
300	68	3	7	2	12	20000	4545	9	1	—	8
400	90	18	2	—	16	30000	6818	3	7	2	12

Note, 1 Firkin is = 1 *s.* 1 *d.* 2 *q.* 12 *pts.* 2 is = 2 *s.* 3 *d.* 1 *q.* 2 *pts.* and 3 is = 3 *s.* 4 *d.* 3 *q.* 14 *pts.*

Table XII. *The Duty of Exciſe to be paid by common Brewers within the Bills of Mortality, for Small-Beer.*

N° of Bar.	*l.*	*s.*	*d.*	*q.*	23 *pts.*	Bar.	*l.*	*s.*	*d.*	*q.*	23 *pts.*
1	—	1	1	3	15	35	2	—	6	3	19
2	—	2	3	3	7	36	2	1	8	3	11
3	—	3	5	2	22	37	2	2	10	3	3
4	—	4	7	2	14	38	2	4	—	2	18
5	—	5	9	2	6	39	2	5	2	2	10
6	—	6	11	1	21	40	2	6	4	2	2
7	—	8	1	1	13	41	2	7	6	1	17
8	—	9	3	1	5	42	2	8	8	1	9
9	—	10	5	—	20	43	2	9	10	1	1
10	—	11	7	—	12	44	2	11	—	—	16
11	—	12	9	—	4	45	2	12	2	—	8
12	—	13	10	3	19	46	2	13	4	—	—
13	—	15	—	3	11	47	2	14	5	3	15
14	—	16	2	3	3	48	2	15	7	3	7
15	—	17	4	2	18	49	2	16	9	2	22
16	—	19	6	2	10	50	2	17	11	2	14
17	—	19	8	2	2	51	2	19	1	2	6
18	1	—	10	1	17	52	3	—	3	1	21
19	1	2	—	1	9	53	3	1	5	1	13
20	1	3	2	1	1	54	3	2	7	1	5
21	1	4	4	—	16	55	3	3	9	—	20
22	1	5	6	—	8	56	3	4	11	—	12
23	1	6	8	—	—	57	3	6	1	—	4
24	1	7	9	3	15	58	3	7	2	3	19
25	1	8	11	3	7	59	3	8	4	3	11
26	1	10	1	2	22	60	3	9	6	3	3
27	1	11	3	2	14	61	3	10	8	2	18
28	1	12	5	2	6	62	3	11	10	2	10
29	1	13	7	1	21	63	3	13	—	2	2
30	1	14	9	1	13	64	3	14	2	1	17
31	1	15	11	1	5	65	3	15	4	1	9
32	1	17	1	—	20	66	3	16	6	1	1
33	1	18	3	—	12	67	3	17	8	—	16
34	1	19	5	—	4	68	3	18	10	—	8

The Duty of Exciſe to be paid by common Brewers within the Bills of Mortality, for Small-Beer, continued.

N° of Bar.	*l.*	*s.*	*d.*	*q.*	23 *pts.*	Bar.	*l.*	*s.*	*d.*	*q.*	23 *pts.*
69	4	—	—	—	—	500	28	19	8	2	2
70	4	1	1	3	15	600	34	15	7	3	7
71	4	2	3	3	7	700	40	11	7	—	12
72	4	3	5	2	22	800	46	7	6	1	17
73	4	4	7	2	14	900	52	3	5	2	22
74	4	5	9	2	6	1000	57	19	5	—	4
75	4	6	11	1	21	2000	115	18	10	—	8
76	4	8	1	1	13	3000	173	18	3	—	12
77	4	9	3	1	5	4000	231	17	8	—	16
78	4	10	5	—	20	5000	289	17	1	—	20
79	4	11	7	—	12	6000	347	16	6	1	1
80	4	12	9	—	4	7000	405	15	11	1	5
90	5	4	4	—	16	8000	463	15	4	1	9
100	5	15	11	1	5	9000	521	14	9	1	13
200	11	11	10	2	10	10000	579	14	2	1	17
300	17	7	9	3	15	20000	1159	8	4	3	11
400	23	3	9	—	20	30000	1739	2	7	1	5

Note, 1 Firkin is 3*d.* 1 *q.* 21 *pts.* 2 is 6*d.* 3 *q.* 19 *pts.* and 3 is 10*d.* 1 *q.* 17 *pts.*

The

The Calculation and Uſe of the Tables in this Supplement.

TABLE I.

The Conſtruction and Uſe of this Table.

THE Duty on Strong-Beer and Ale for common Brewers, is 5 *s.* *per* Barrel; but out of that they are allowed, without the Bills of Mortality, two Barrels and an half in every 23 Barrels. So that as 23 : is to 12 *s.* 6 *d.* :: So is 1 to 6 *d.* 2 *q.* $\frac{2}{23}$, the Allowance *per* Barrel; which being therefore deducted from 5 *s.* the Remainder is 4 *s.* 5 *d.* 1 *q.* $\frac{21}{23}$, to be paid for each Barrel: At which Rate,

The Duty, for Example,	*l.*	*s.*	*d.*	*q.*
of 10087 Barrels, is =	2247	12	11	0 $\frac{20}{23}$
For the Duty to be paid for 10000, is =	2228	5	2	2 $\frac{10}{23}$
And for 87 — =	19	7	8	2 $\frac{10}{23}$
Sum or Anſwer	2247	12	11	0 $\frac{20}{23}$

Table II. *The Conſtruction and Uſe of this Table.*

The Duty on Small-Beer for common Brewers is 1 *s.* 4 *d.* *per* Barrel; but they are allow'd out of that 2 and a half in 23 Barrels, as for Strong-Beer without the Weekly-Bills. So that, as 23 : is to 3 *s.* 4 *d.* :: So is 1 : to 1 *d.* 2 *q.* $\frac{22}{23}$ of *qr.* which being deducted from 1 *s.* 4 *d.* the Remainder is = 1 *s.* 2 *d.* 1 *q.* $\frac{1}{23}$, the

Duty to be paid *per* Barrel for Small-Beer by common Brewers. The Uſe is as before.

Tables III and IV. need no Explanation, there being inſerted only the Duty (and no Allowance) to be paid by Victuallers; and the Uſe is very eaſy: For Example, the Duty of 10073 Barrels is 2518*l.* 5*s.* for Strong-Beer: As,

	l.	*s.*	*d.*
The Duty of 10000 (adding a 0 to that of 1000) is =	2500	0	0
And the Duty for 73 Barrels is =	18	5	0

And for Small-Beer by Victuallers, for the ſame Quantity is,

	l.	*s.*	*d.*
For 10000 (*66 l.* 13 *s.* 4 *d.* the Duty for 1000, multiply'd by 10) is	*666*	13	4
And for 73 Barrels — =	4	17	4
Sum or Duty =	*671*	10	8

More Uſe of Table IV. after Table IX. its Uſe.

Table V. *The Conſtruction and Uſe of this Table*, which is the Duty to be paid for Malt in the Ciſtern, or within 30 Hours after it is thrown out; the Allowance of 4 Buſhel in 20 being deducted out of *6 d. per* Buſhel, For the Allowance, I ſay, if 20 Buſhel allow 2 *s.* what 1 Buſhel?

Multiply and divide, and the Anſwer is 1 *d.* 0 *qr.* $\frac{16}{20}$, or $\frac{4}{5}$ of a *qr.*

And *6 d.* leſs 1 *d.* 0 *q.* $\frac{4}{5}$, is = 4 *d.* 3 *q.* $\frac{1}{5}$, the Duty to be paid; at which Rate 8000 and 77 Buſhel payeth *160 l.* and 1 *l.* 10 *s.* 9 *d.* 2 *q.* $\frac{2}{5}$, or 8077 Buſhel payeth *161 l.* 10 *s.* 9 *d.* 2 *q.* $\frac{2}{5}$.

Table VI. *The Conſtruction and Uſe of this Table*, which is the Duty to be paid for Malt 30 Hours or more after thrown out of the Ciſtern, and before dry'd; the Allowance being 10 Buſhel in 20, or 3 *d.* out of 6 *d.* needs no farther Explanation: And the Duty (by the Table) of 2085 Buſhel,

l. *s.* *d.*
is for 2000 Buſhel = 25 0 0 } Sum or Anſw.
And for 85 = 1 1 3 } 26 *l.* 1 *s.* 3 *d.*

2dly, Or for printed Linnen at 3 *d. per* ſquare Yard, the Duty by the ſame Table VIth, is of 20795 Yards. Thus,

l. *s.* *d.*
For 10000 125 *l.* double that is = 250 0 0
700 *l.* is - - - - - - 8 15 0
95 is - - - - - - - 1 3 9
The Sum or Anſwer = 259 18 9

Table VII. needs no Conſtruction; the Uſe is, ſuppoſe I would find the Duty of 30497 Ounces of Plate, at 6 *d. per* Ounce.

l. *s.* *d.*
30000 is = 750 0 0 } *l.* *s.* *d.*
490 is = 12 5 0 } = 762 8 6 the Sum
7 Sixpences is 0 3 6 } or Anſwer.

Table VIII. Is alſo very eaſy to uſe; for it ſhews that the Duty for 32076 Pounds of Soap, at 1 *d.* ½ *per* Pound, is 200 *l.* 9 *s.* 6 *d.*

l. *s.* *d.*
As for 30000 - 187 10 0 } Sum or Anſwer
2000 - 12 10 0 } 200 *l.* 9 *s.* 6 *d.*
76 - - 0 9 6 }

Table IX. Its Uſe is likewiſe eaſy, for by it the Duty of 8979 Pound of Tallow-Candles, at 1 *d. per* Pound is 37 *l.* 8 *s.* 3 *d.*

For

For 2000 multiply'd by 4 = 8000 is *l.*	33	6	8
970 is	4	0	1
and 9 is	0	0	9
Sum or Anſwer	37	8	3

Second Uſe of the IVth Table may be to compute the Duty on Wax-Candles at 8 *d. per* Pound; which is by taking half of what is in that IVth Table. As for Example, the Duty of 3959 Pound, at 8 *d.*

	l.	*s.*	*d.*
3000 in the IVth Table is	200	0	0
950 is — — —	63	6	8
9 is — — —	0	12	0

Sum is = 263 18 8

Half is the Anſwer = 131 19 4

Table X. *The Calculation of this Table and Uſe*, which is the Duty of Exciſe to be paid for Strong Beer by common Brewers within the Weekly-Bills of Mortality, which is 5 *s. per* Barrel, (or 36 Gal.) but out of that they are allow'd 3 Barrels in every 23. So that,

As 23 : is to 15 *s.* :: So is 1 to 7 *d.* 3 *q.* $\frac{7}{23}$ *pts.* the Allowance *per* Barrel; which deducted from 5 *s.* there reſts 4 *s.* 4 *d.* 0 *q.* $\frac{16}{23}$ Parts to be paid *per* Barrel. So that by this Table the Duty to be paid for 14773 Barrels is,

	l.	*s.*	*d.*	*q.*	23 *pts.*
For 10000 Barrels —	2173	18	3	3	12
4000 ———	869	11	3	2	14
700 ———	152	3	5	2	22
73 ———	15	17	4	2	18
Sums = 14773 ———	3211	10	5	0	20

Table XI. *The Calculation, Conſtruction, and Uſe of this Table*, which is the Duty of Exciſe to be paid for Ale by common Brewers within the

Bills of Mortality, which is 5 *s. per* Barrel; (or 32 Gallons) but out of that they are allowed 2 Barrels in every 22. So that,

As 22 : to 10 *s.* : : So is 1 : to 5 *d.* 1 *q.* 18 22 *pts.* = the Allowance *per* Barrel; which deducted from 5 *s.* there reſts 4 *s.* 0 *d.* 2 *q.* 4, 22 Parts, to be paid *per* Barrel; at which Rate the Duty of 5079 Barrels is.

	l.	*s.*	*d.*		22 *pts*
For 5000 Barrels ——	1136	7	3	1	2
79 ——	17	19	1	0	8
Sums = 5079 ——	1154	6	4	1	10

Table XII. *The Conſtruction and Calculation of this Table,* which is the Duty of Exciſe to be paid for Small Beer by commom Brewers within the Bills of Mortality, which is 1 *s.* 4 *d. per* Barrel; (or 36 Gallons) but out of that they are allow'd 3 Barrels in 23, (as for Strong Beer;) So the Allowance is *per* Barrel thus,

As 23 : to 4 *s.* :: So 1 : to 2 *d.* 0 *q.* 8, 23 *pts.* of a Farthing; which deducted from 1 *s.* 4 *d.* the Remainder is 1 *s.* 1 *d.* 3 *q.* 15 *pts.* to be paid for 1 Barrel, and for any Number at that Rate, as *per* the Table, by which 20971 Balrels pays,

	l.	*s.*	*d.*	*q.*	23 *pts.*
20000 ——	1159	8	4	3	11
900 ——	52	3	5	2	22
71 ——	4	2	3	3	7
Sums = 20971 ——	1215	14	2	1	17

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The End of the SUPPLEMENT.